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Numerical Analysis of Darcy-Forchheimer Flow and Heat Transfer Over a Stretching Sheet with a Uniform Heat Source

Puspanjali Mahapatra[1], Kharabela Swain[2], Tusar Parida[1,*]

[1] Department of Mathematics, Silicon University, Bhubaneswar-751024, India; puspanjali.mahapatra@gmail.com; kharabela1983@gmail.com.
[2] Department of Mathematics, Gandhi Institute For Technology (GIFT), Bhubaneswar-752054, India; tusar.parida@silicon.ac.in.

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Abstract

The analysis explores the Darcy-Forchheimer flow and heat transfer of Maxwell fluid over a vertical stretching sheet uniform heat source/sink. The governing Partial Differential Equations (PDEs) are converted using similar transformations into nonlinear Ordinary Differential Equations (ODEs). The resulting ODEs are solved numerically using the Runge-Kutta fourth-order method along with the shooting technique. The outcomes of relevant parameters on velocity, temperature, skin friction coefficient, and local Nusselt number are illustrated in graphs and tables. The local inertia parameter, which is responsible for inertia drag, is found to reduce the fluid velocity, but an adverse effect is observed on the temperature field.

Keywords: Darcy-Forchheimer flow, Heat transfer, Vertical stretching sheet, Uniform heat source/sink.

1 | Introduction

The flow problem in the boundary layer induced by a continuously moving or stretching surface is essential in many manufacturing processes. In industry, polymer sheets and filaments are manufactured by a continuous extrusion of the polymer from a die to a windup roller located at a finite distance away from a die. The thin polymer sheet constitutes a continuously moving surface with a non-uniform velocity through

Corresponding Author: puspanjali.mahapatra@gmail.com

an ambient fluid or fluid with some prescribed velocity. Crane [1] examined the boundary layer flow caused by stretching sheet.

The flow and heat transfer through a porous media has received much attention in the past few years due to its various practical applications in geothermal and oil reservoir engineering as well as other geophysical and astrophysical studies. Nayak et al. [2] have studied viscoelastic fluid over a stretching sheet. Dessie and Kishan [3] have incorporated viscous dissipation and heat source/sink on the heat transfer flow of a fluid over a stretching sheet. Swain et al. [4] have studied the effect of variable viscosity and thermal conductivity on MHD heat and mass transfer over a stretching sheet. Mahdy and Chamkha [5] have considered viscous dissipation and chemical reactions on mixed convective Darcy-Forchheimer fluid flow. All these studies have been confined to a porous medium.

A new dimension is added to the study of flow and heat transfer in a viscous fluid over a stretching surface by considering the effect of thermal radiation. The thermal radiation effect might significantly affect the heat transfer process in polymer processing industries. The quality of the final product depends, to a great extent, on heat control factors. The knowledge of radiative heat transfer in the system can perhaps lead to a desired product with the desired characteristics. Many processes in engineering areas occur at high temperatures, and knowledge of radiation heat transfer is significant for the design of the pertinent equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Makinde [6] has examined the effect of thermal radiation past a moving vertical porous plate. Golafshan and Rahimi [7] have studied the effect of radiation on the mixed convection stagnation point flow of nanofluid over a vertical stretching sheet. Slip flow analysis on MHD nanofluid over a vertical stretching sheet with higher order chemical reaction is carried out by Swain et al. [8]. Hayat et al. [9] analyzed the heat and mass transfer of Darcy-Forchheimer flow with a chemical reaction. Arifin et al. [10] studied the stagnation point flow in a Darcy-Forchheimer porous medium over a shrinking sheet with slip boundary conditions. Uddin et al. [11] analyzed numerically the Darcy-Forchheimer flow of Sisko nanomaterial with nonlinear thermal radiation. Rasool et al. [12] carried out an MHD Darcy-Forchheimer nanofluid flow over a nonlinear stretching sheet.

This study aims to put up a mathematical model for Darcy-Forchheimerboundary layer flow and heat transfer past a vertical stretching sheet with a non-uniform heat source/sink. The governing Partial Differential Equations (PDEs) are transferred into nonlinear Ordinary Differential Equations (ODEs) using similarity transformations, which are then solved by the Runge-Kutta fourth-order method and shooting technique. The effects of different parameters are presented through graphs and illustrated in detail. Further, the shearing stress and the rate of heat transfer at the plate have been computed via tables and discussed in detail.

2 | Mathematical Formulation

We consider a steady two-dimensional boundary layer flow and heat transfer over a vertical stretching sheet in the presence of a uniform heat source/sink. The flow is assumed to be in the x-direction, chosen along the sheet and the axis perpendicular to it. Let u and v are the tangential and normal velocities of the fluid, respectively. The differential equations of fluid motion are based on Forchheimer, which accounts for the drag exerted by the porous media in the study of porous media flow analysis. The governing equations with boundary layer conditions under Boussinesq's approximation are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u-\frac{\upsilon}{Kp^*}u-\frac{c_b}{\sqrt{Kp^*}}u^2. \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q^*}{\rho c_p}\left(T-T_\infty\right). \tag{3}$$

$$y=0:u=u_w\left(x\right)=ax, v=0,-k\frac{\partial T}{\partial y}=h\left(T_w-T_\infty\right),$$
$$y\to\infty:u=0,T\to T_\infty, \tag{4}$$

where u,v are velocity components in x and y directions respectively, B_0 is the magnetic field strength, υ is the kinematic viscosity, σ is the electrical conductivity, ρ is the density, k is the thermal conductivity, T is the temperature, T_∞ is the ambient temperature of the fluid, c_p is the specific heat, c_b is the drag coefficient, Q^* is the heat source/sink coefficient, Kp^* is the permeability of the medium, $a\left(>0\right)$ is a constant, and h is the heat transfer coefficient.

By using the following similarity transformations and non-dimensional variables

$$\eta=\sqrt{\frac{a}{\upsilon}}y, u=axf'\left(\eta\right), v=-\sqrt{\upsilon a}f\left(\eta\right), \theta\left(\eta\right)=\frac{T-T_\infty}{T_w-T_\infty}.$$

The *Eqs. (1)-(4)* can be written as

$$f'''-\left(1+Fr\right)f'^2+ff''-\left(M+Kp\right)f'=0. \tag{5}$$

$$\frac{1}{Pr}\theta''+f\theta'+Q\theta=0. \tag{6}$$

$$\left.\begin{array}{l}\eta=0:f\left(0\right)=0, f'\left(0\right)=1, \theta'\left(0\right)=-Bi\left[1-\theta\left(0\right)\right]\\ \eta\to\infty:f'\left(\infty\right)\to0, \theta\left(\infty\right)\to0\end{array}\right\}, \tag{7}$$

where $M=\frac{\sigma B_0^2}{a\rho}$ is the magnetic parameter, $Kp=\frac{aKp^*}{\upsilon}$ is the porosity parameter, $Fr=\frac{c_b}{x\sqrt{Kp^*}}$ is the local inertia parameter, $Pr=\frac{\mu c_p}{k}$ is the Prandtl number, $Q=\frac{Q^*}{a\rho c_p}$ is the heat source/sink parameter, and $Bi=\frac{h}{k}\sqrt{\frac{\upsilon}{a}}$ is the Biot number.

The skin friction coefficient $\left(C_f\right)$ and local Nusselt number $\left(Nu_x\right)$ are given by $C_f=\frac{2\tau_w}{\rho u_w^2}$ and $Nu_x=\frac{xq_w}{k\left(T_w-T_\infty\right)}$, respectively.

Here, wall shear stress $\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}\Rightarrow C_f\sqrt{Re_x}=-f''\left(0\right)$ and wall heat flux $q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}\Rightarrow\frac{Nu_x}{\sqrt{Re_x}}=-\theta'\left(0\right)$ where $Re_x=\frac{ax^2}{\upsilon}$ is the local Reynolds numbers.

Flow model and coordinate system shown in *Fig*. 1.

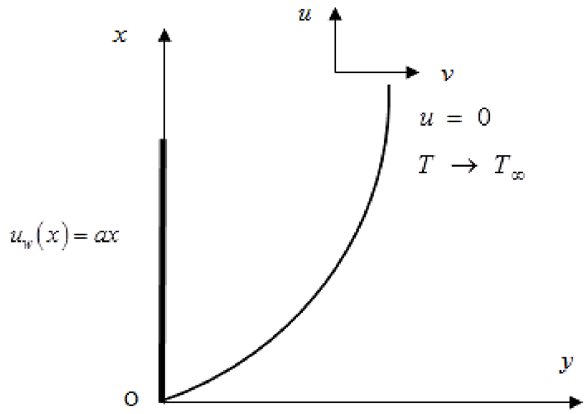

Fig. 1. Flow model and coordinate system.

3 | Results and Discussion

The coupled, nonlinear PDEs in *Eqs. (5)-(7)* are solved by the Runge-Kutta fourth-order method with a shooting technique using MATLAB software with step length $\Delta\eta = 0.01$ and error tolerance 10^{-5}. The validation was accomplished by comparing the present study with that of Khan and Pop [13], as particular case assigning $M = Fr = Kp = Bi = 0$ as shown in *Table 1*. During the discussion, we have fixed the non-dimensional parameters as $M = 0.1, Kp = 0.5, Fr = 1, Q = Bi = 0.1$ and $Pr = 2$ unless otherwise the values are mentioned.

Table 1. Comparison of the values of $-\theta'(0)$.

	$-\theta'(0)$	
Pr	**Khan and Pop [13]**	**Present Study**
0.07	0.0663	0.064295
0.2	0.1691	0.168294
0.7	0.4539	0.451590
2	0.9113	0.910680
7	1.8954	1.894921
20	3.3539	3.353507
70	6.4621	6.461822

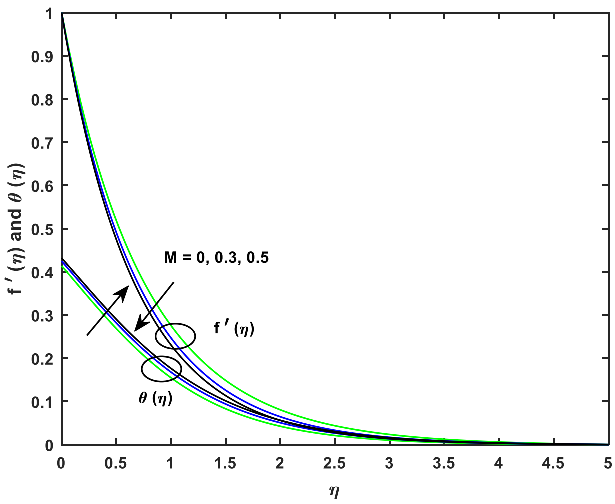

Fig. 2. Influence of M on velocity and temperature profiles.

Fig. 2 shows the effect of magnetic parameter (M) on velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles. An increase in magnetic parameters causes a Lorentz force that works in the opposite direction of the fluid flow. This force tends to slow down the motion of the fluid flow and, consequently, produce more heat in the fluid. Therefore, an increase in M decreases the velocity and increases the fluid's temperature in the flow domain.

Fig. 3 is depicted to study the influence of local inertia parameter (Fr) on velocity and temperature profiles in the presence $(\mathrm{Kp}=0.5)$ and absence $(\mathrm{Kp}=0)$ of a porous matrix. It is elucidated that on increasing in (Fr) the fluid velocity decreases. In the case of porous spaces with bigger pore sizes, the Forchheimer number accounts for the inertia effects due to porous medium and pressure drop disturbed by fluid-solid interaction, which dominates the viscous interference. Thus, an increase in the local inertia parameter causes a more excellent resistance to the flow; hence, the fluid velocity decreases and enhances the fluid's temperature since more heat is generated due to the porous medium.

Fig. 4 displayed to analyze the impact of heat source/sink parameter (Q) on temperature distribution. It is evident from this figure that the thickness of the thermal boundary layer increases with increasing heat source/sink parameter (Q).

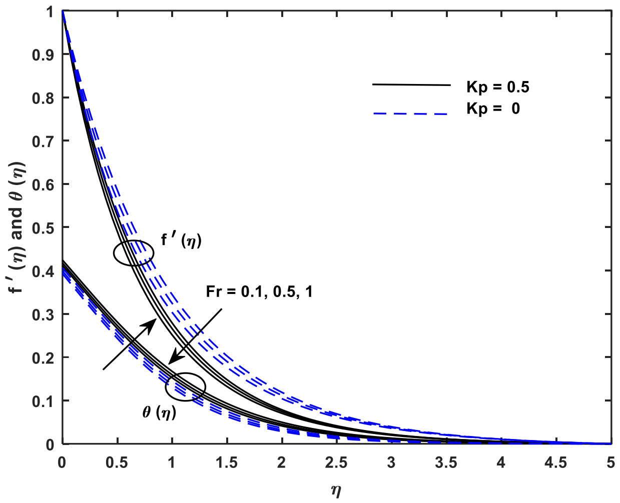

Fig. 3. Influences of Fr and Kp on velocity and temperature profiles.

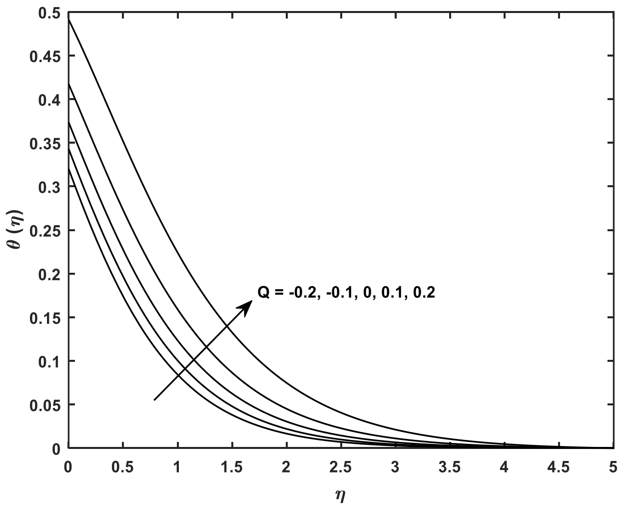

Fig. 4. Influence of Q on temperature profile.

Table 2. Values of skin friction coefficient $-f''(0)$ and Nusselt number $-\theta'(0)$ when Kp=0.5 and Pr=2.

M	Fr	Bi	Q	$-f''(0)$	$-\theta'(0)$
0.1	0.1	0.1	0.1	1.290355	0.087724
0.5				1.620293	0.095979
1.0				1.735964	0.096853
	0.5			1.780571	0.094248
	1.0			1.928377	0.093136
		0.3		1.928348	0.272901
		0.5		1.928347	0.367355
			0.3	1.919284	0.343609
			0.5	1.901410	0.308226

Table 2 is computed to observe the impacts of important physical parameters M, Fr, Bi and Q on skin friction coefficient and Nusselt number. The skin friction coefficient is enhanced with increasing values of M and Fr whereas slightly decreases with higher values of Q. On the other hand, the local Nusselt number is getting enhanced on increasing values of M and Bi whereas decreases on increasing either of Fr and Q. These results are well supported by Seth et al. [14].

4 | Conclusion

The present article focuses on the effect of a uniform heat source/sink on magnetohydrodynamic flow and heat transfer of an electrically conducting fluid over a vertical stretching sheet in a non-Darcy porous medium. The equations of the stated flow are solved numerically using an effective shooting technique, and graphs are plotted using obtained numerical values. We have compared the present study results with the existing results, and they agree with the previous results. The following conclusions can be drawn from the present study:

I. Increasing magnetic parameter values decreases the fluid's velocity but enhances the thermal resistance, boosting the temperature profile.

II. The local inertia parameter (Forchheimer parameter), responsible for inertia drag, reduces the fluid velocity, but an adverse effect is observed in the temperature field.

III. Biot number and Heat source/sink parameters increasingly affect temperature distribution.

IV. The magnetic and local inertia parameters have the same effects on the skin friction coefficient but have opposite effects on the local Nusselt number.

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