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Revolutionary Analysis of Pointwise Stationary Fluid Flow Approximation in Non-Stationary M/D/1 Queue with IoT Applications

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Abstract

For a long-standing solution via simulation, this study presents the first-ever analytic modelling for the Pointwise Stationary Fluid Flow Approximation (PSFFA) model of the non-stationary $M/D/1$ queueing system. This is accomplished by putting out the constant ratio β (Ismail's ratio), which provides a precise analytical answer and links the time-dependent mean arrival and mean service rates. We then do a numerical analysis of the stability dynamics of the time-varying $M/D/1$ queueing system with respect to time β and the queueing parameters. Applications of Pointwise Fluid Flow Approximation (PSFFA) to the Internet of Things are given. A summary and recommendations for further research round out the paper.

Keywords: Time varying $M/D/1$ queue, Pointwise, Fluid flow approximation, Internet of Things.

1 | Introduction

There are barely few works in the topic of transient/non-stationary analysis that falls into the categories of simulative approaches, combined with other applied strategies, covering a range of methods for researching systems that undergo temporal change, such as using simulations, examining non-stationary phenomena, and examining transient behaviour. In certain situations, a closed form statement for the analysis of non-stationary queueing systems can be obtained by mathematical transformations.

On the other hand, computing these expressions can be difficult. The focus now is on quantitatively determining the transient behaviour of these types of systems rather than on developing closed form formulas. The current exposition helps to answer the long-standing unsolved issue of acquiring the time-varying $M/D/1$ queueing system's state variable for the first time ever.

The sequence that follows demonstrates how this paper is structured.

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I. Pointwise Stationary Fluid Flow Approximation (PSFFA).

II. Solving PSFFA model of the non-stationary $M/D/1$ queue.

III. PSFFA applications to IoT.

IV. Closing remarks with next phase of research.

2 | Pointwise Stationary Fluid Flow Approximation (PSFFA)

Let $f_{in}(t)$ and $f_{out}(t)$ serve as the temporal flow in, and flow out, respectively. Thus,

$$\frac{dx(t)}{dt} = x^{\cdot}(t) = -f_{out}(t) + f_{in}(t), x(t) \text{as the state variable.} \quad (1)$$

$f_{out}(t)$ links server utilization, $\rho(t)$ and the time-dependent mean service rate, $\mu(t)$ by

$$f_{out}(t) = \mu(t)\rho(t). \quad (2)$$

For an infite queue waiting space:

$$f_{in}(t) = \text{Mean arrival rate} = \lambda(t). \quad (3)$$

Thus (2) rewrites to:

$$x^{\cdot}(t) = -\mu(t)\rho(t) + \lambda(t)\,, \; 1 > \rho(t) = \frac{\lambda(t)}{\mu(t)} > 0. \quad (4)$$

The stability phase of (4) (i.e., $x^{\cdot}(t) = 0$), implies:

$$x = G_1(\rho). \quad (5)$$

The numerical invertibility of $G_1(\rho)$, yields

$$\rho = G_1^{-1}(x). \quad (6)$$

Hence,

$$x^{\cdot}(t) = -\mu(t)\left(G_1^{-1}(x(t))\right) + \lambda(t). \quad (7)$$

The $M/D/1$ queueing system is made of Poisson arrival, one exponential (Poisson) server, FIFO (First-In-First-Out). Thus, $M/D/1$ queueing system's -G_1 (c.f.,[1]) reads:

$$G_1(x) = \left((x+1) - \sqrt{(x^2+1)}\right). \quad (8)$$

Accordingly, the resulting PSFFA model is:

$$x^{\cdot} = -\mu\left((x+1) - \sqrt{(x^2+1)}\right) + \lambda. \quad (9)$$

Non-staionary queues' life example [2] is depicted by *Fig. 1*.

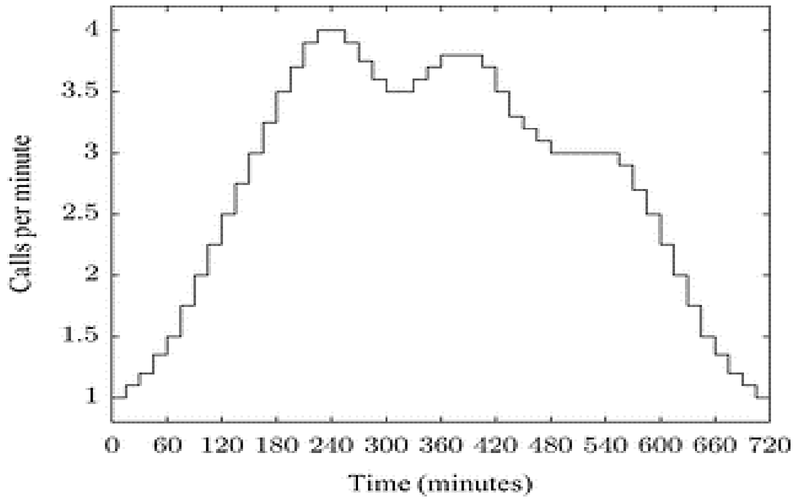


Fig. 1. Non-staionary queues' life example [2].

3 | Solving The Non-Stationary M/D/1 Queueing System's Psffa

Theorem 1. The analytic solution of *Eq. (9)*, using Ismail's ration, β is

$$\left(x-\beta(x+1)\right)^{\frac{1}{(1-\beta)^2}} = \gamma e^{\left[\frac{(x+1)}{(1-\beta)} - \int \mu(t)dt\right]} \quad ,\beta = \frac{\lambda(t)}{\mu(t)}. \quad \textbf{(10)}$$

Proof: We have

$$x^{\cdot} = -\mu\left((x+1) - \sqrt{(x^2+1)}\right) + \lambda. \quad \textbf{(c.f., (9))}$$

Let, then $x^{\cdot} = -y^{\cdot}$ cothy cschy. Setting, $\beta = \frac{\lambda(t)}{\mu(t)}$. Thus, we have

$$-y^{\cdot} \text{ cothy cschy} = -\mu\left((1+\text{cschy}) - \text{cothy}\right) + \lambda = -\mu[(1+\text{cschy}) - \text{cothy}) - \beta]. \quad \textbf{(11)}$$

Therefore, we have

$$\frac{-\text{cothy cschydy}}{[(1+\text{cschy}) - \text{cothy}) - \beta]} = -\mu dt = \frac{\text{coshydy}}{[-(1+\text{sinhy} - \text{coshy}) + \beta\text{sinhy}]\text{sinhy}}. \quad \textbf{(12)}$$

$$\frac{\frac{2}{\beta}(e^{3y}+e^{y})dy}{\left[e^{2y} - \frac{e^{y}}{\beta} + (\frac{2}{\beta} - 1)\right](e^{2y}-1)} = -\mu dt.$$

$$e^{2y} - \frac{e^{y}}{\beta} + \left(\frac{2}{\beta} - 1\right) = 0 \Rightarrow e^{y} = a, b,$$

where

$$a = \frac{\left(\frac{1}{\beta} + \sqrt{(\frac{1}{\beta^2} - \frac{8}{\beta} + 4)}\right)}{2}, b = \frac{\left(\frac{1}{\beta} - \sqrt{(\frac{1}{\beta^2} - \frac{8}{\beta} + 4)}\right)}{2}.$$

Let

$$\frac{\frac{2}{\beta}(e^{3y}+e^{y})}{\left[e^{2y} - \frac{e^{y}}{\beta} + (\frac{2}{\beta} - 1)\right](e^{2y}-1)} = \frac{A}{(e^{y}-a)} + \frac{B}{(e^{y}-b)} + \frac{C}{(e^{y}-1)} + \frac{D}{(e^{y}+1)}. \quad \textbf{(13)}$$

Hence, it is implied that

$$A + B + C + D = \frac{2}{\beta} \Longrightarrow A = \frac{2}{\beta} - (B + C + D).$$

$$\therefore B = \frac{(1-a)(C-D)}{\left(\frac{2}{\beta}+a\right)}. \tag{14}$$

Thus, D equals

$$\left(\frac{\frac{2b}{\beta} + a\left(ab - \frac{b\left(1+\frac{2}{\beta}\right)}{\left(\frac{2}{\beta}+a\right)} + a\left[\frac{(1-a)}{\left(\frac{2}{\beta}+a\right)}\right]\right)\left[\frac{\frac{4}{\beta}\left(\frac{2}{\beta}+a\right)}{\left(\frac{2}{\beta}\right) + a + [ab-(a+b)]\left(\frac{2}{\beta}+a\right)}\right]}{\left(\left(ab + b\left[\frac{\left(1+\frac{2}{\beta}+2a\right)}{\left(\frac{2}{\beta}+a\right)}\right] + a\left[\frac{(1-a)}{\left(\frac{2}{\beta}+a\right)}\right]\right) + a\left(ab - \frac{b\left(1+\frac{2}{\beta}\right)}{\left(\frac{2}{\beta}+a\right)} + a\left[\frac{(1-a)}{\left(\frac{2}{\beta}+a\right)}\right]\right)\left[\frac{\left(1+\frac{2}{\beta}+2a\right) - (ab-(a+b))\left(\frac{2}{\beta}+a\right)}{\left(\frac{2}{\beta}\right) + a + [ab-(a+b)]\left(\frac{2}{\beta}+a\right)}\right]\right)}\right). \tag{15}$$

$$\left[\left(1+\frac{2}{\beta}+2a\right) - \left(ab-(a+b)\right)\left(\frac{2}{\beta}+a\right)\right] = \tag{16}$$

$$C = \frac{4}{\beta}\left(\frac{2}{\beta}+a\right) - \frac{\frac{2b}{\beta}\left[\left(1+\frac{2}{\beta}+2a\right) - (ab-(a+b))\left(\frac{2}{\beta}+a\right)\right] + \frac{4a}{\beta}\left(\frac{2}{\beta}+a\right)\Big]\left(ab - \frac{b\left(1+\frac{2}{\beta}\right)}{\left(\frac{2}{\beta}+a\right)} + a\left[\frac{(1-a)}{\left(\frac{2}{\beta}+a\right)}\right]\right)}{\frac{\left(ab + b\left[\frac{b\left(1+\frac{2}{\beta}+2a\right)+a(1-a)}{\left(\frac{2}{\beta}+a\right)}\right]\frac{2}{\beta} + a + [ab-(a+b)]\left(\frac{2}{\beta}+a\right) + a\left(ab - \frac{b\left(1+\frac{2}{\beta}\right)}{\left(\frac{2}{\beta}+a\right)} + a\left[\frac{(1-a)}{\left(\frac{2}{\beta}+a\right)}\right]\right)\left[\left(1+\frac{2}{\beta}+2a\right) - (ab-(a+b))\left(\frac{2}{\beta}+a\right)\right]}{\left(\frac{2}{\beta}\right) + a + [ab-(a+b)]\left(\frac{2}{\beta}+a\right)}}).$$

$$A = \frac{2}{\beta} - \frac{\left(1+\frac{2}{\beta}\right)C}{\left(\frac{2}{\beta}+a\right)} - \frac{\left(1+\frac{2}{\beta}+2a\right)D}{\left(\frac{2}{\beta}+a\right)}, \quad B = \frac{(1-a)(C-D)}{\left(\frac{2}{\beta}+a\right)}. \tag{17}$$

This finally solves the complicated mathematical computations to obtain

$$\frac{|(1-ae^{-y})|^{A}|(1-be^{-y})|^{B}|(1-e^{-y})|^{C}}{(1+e^{-y})^{D}} = \eta e^{-\int \mu dt}, \eta > 0. \tag{18}$$

This transforms to the final required closed form solution:

$$\frac{|\left(1-ae^{-\operatorname{csch}^{-1}(x)}\right)|^{A}|\left(1-be^{-\operatorname{csch}^{-1}(x)}\right)|^{B}|\left(1-e^{-\operatorname{csch}^{-1}(x)}\right)|^{C}}{\left(1+e^{-\operatorname{csch}^{-1}(x)}\right)^{D}} = \eta e^{-\int \mu dt}. \tag{19}$$

Since,

$$\operatorname{csch}^{-1}(x) = \ln\left(\frac{1+\sqrt{1+x^2}}{x}\right). \tag{20}$$

With the domain of real line with zero removed Thus, one gets

$$\frac{\left(\left|\left(1-\frac{ax}{1+\sqrt{1+x^2}}\right)\right|\right)^{A}\left(\left|\left(1-\frac{bx}{1+\sqrt{1+x^2}}\right)\right|\right)^{B}\left(\left|\left(1-\frac{x}{1+\sqrt{(1+x^2)}}\right)\right|\right)^{C}}{\left(1+\frac{x}{1+\sqrt{(1+x^2)}}\right)^{D}} = \eta e^{-\int \mu dt}. \tag{21}$$

Corollary 1. As $x(t) \to 0$, we have

$$\eta e^{-\int \mu dt} = \lim_{x(t)\to 0} \frac{\left(|1-\frac{ax}{1+\sqrt{1+x^2}}|\right)^{\mathbf{A}}\left(|\left(1-\frac{bx}{1+\sqrt{(1+x^2)}}\right)|\right)^{\mathbf{B}}\left(|1-\frac{x}{1+\sqrt{1+x^2}}|\right)^{C}}{\left(1+\frac{x}{1+\sqrt{(1+x^2)}}\right)^{D}} = 1.$$

Implying

$$\int \mu dt = \ln\gamma. \quad \textbf{(22)}$$

Corollary 2. As $x(t) \to \infty$, we have

$$\eta e^{-\int \mu dt} = \lim_{x(t)\to \infty} \frac{\left(|1-\frac{ax}{1+\sqrt{1+x^2}}|\right)^{\mathbf{A}}\left(|\left(1-\frac{bx}{1+\sqrt{(1+x^2)}}\right)|\right)^{\mathbf{B}}\left(|1-\frac{x}{1+\sqrt{1+x^2}}|\right)^{C}}{\left(1+\frac{x}{1+\sqrt{(1+x^2)}}\right)^{D}}$$

$$= \lim_{x(t)\to \infty} \frac{\left(|1-\frac{a}{\frac{1}{x}+\sqrt{1+\left(\frac{1}{x}\right)^2}}|\right)^{\mathbf{A}}}{\left(1+\frac{1}{\frac{1}{x}+\sqrt{(1+(\frac{1}{x})^2)}}\right)^{D}}\left(\left(|\left(1-\frac{b}{\frac{1}{x}+\sqrt{(1+(\frac{1}{x})^2)}}\right)|\right)^{\mathbf{B}}\left(|1-\frac{1}{\frac{1}{x}+\sqrt{1+\left(\frac{1}{x}\right)^2}}|\right)^{C}\right) \quad \textbf{(23)}$$

$$= \frac{(|1-a|)^{\mathbf{A}}(|(1-b)|)^{\mathbf{B}}\left(|1-\frac{1}{\frac{1}{x}+\sqrt{1+\left(\frac{1}{x}\right)^2}}|\right)^{C}}{\left(1+\frac{1}{\frac{1}{x}+\sqrt{(1+(\frac{1}{x})^2)}}\right)^{D}} = 0 \ \to \int \mu dt \to \infty.$$

Numerical experiment one

Let $\beta = 2$, then $a = 0.5$, $b = 0$, $\eta = 1, \mu(t) = t, A = -7.923076923, B = 1.846153846, C = 6, D = 0.4615384615$.

We have

$$\sqrt{2}\frac{\left(\left|\left(1-\frac{0.5x}{1+\sqrt{1+x^2}}\right)\right|\right)^{-7.923076923}\left(\left|\left(1-\frac{x}{1+\sqrt{1+x^2}}\right)\right|\right)^{6}}{\left(1+\frac{x}{1+\sqrt{1+x^2}}\right)^{0.4615384615}}] = t.$$

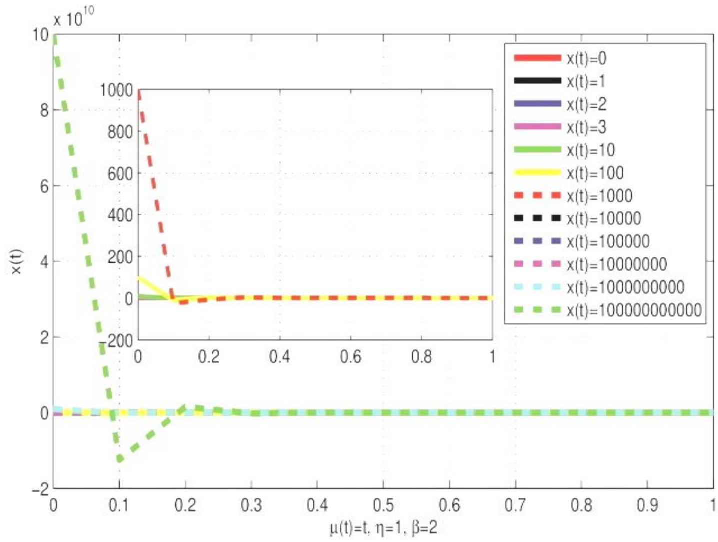


Fig. 2. A new phenomenon for queuing theorists.

Fig. 2 shows a new phenomenon for queuing theorists. The possibility that time will converge to a certain value for sufficiently large number in the time varying $M/D/1$ queuing system. This is for an increasing temporal meanservice rate. This shows that as the time varying $M/D/1$ queuing system's state variable becomes sufficiently large, time vanishes.

Numerical experiment two

Let $\beta = 2$, then $a = 0.5$, $b = 0$, $\eta = 1$, $\mu(t) = \frac{1}{t}$, $A = -7.923076923$, $B = 1.846153846$, $C = 6$, $D = 0.4615384615$.

We have

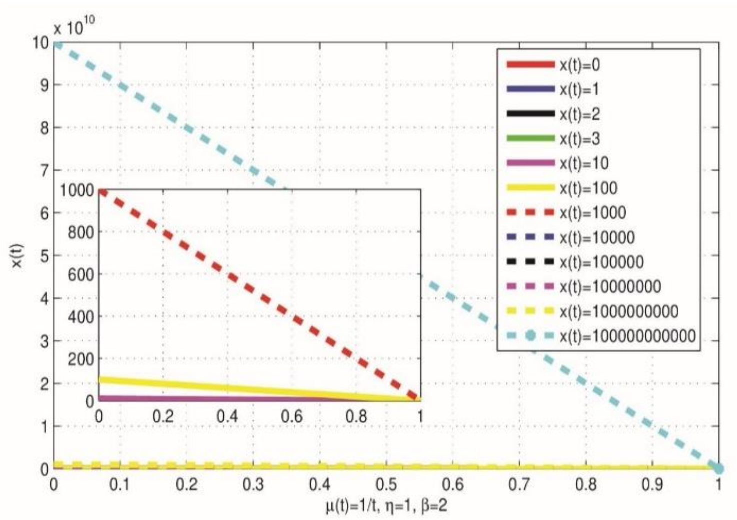


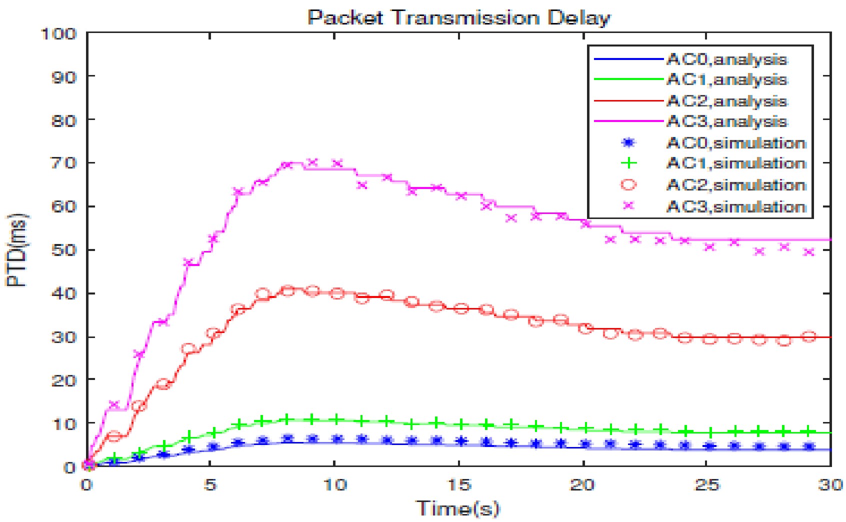
Fig. 3. A new phenomenon for queuing theorists.

Fig. 3 visualizes a new phenomenon to queuing theorist. The possibility that time will converge to a certain value for sufficiently large number in the time varying $M/D/1$ queuing system. This is for a decreasing temporal mean service rate.

4 | Some Psffa Applications To Iot

By describing how vehicles in a platoon use 802.11p communication to exchange messages and change their movement characteristics at intersections, a time-dependent model for assessing the platooning communications' effectiveness at intersections phase was thoroughly investigated [3].

The model evaluates the effectiveness of platooning communications and addresses potential safety concerns by considering variables including vehicle behaviors, traffic signals, and the changing connectivity among vehicles. The authors [3] used PSFFA to describe the transmission queue's dynamic behaviour in platooning communications. They also create models that characterize the continuous backoff freeze and four Access Categories (ACs) of 802.11p as they relate to the time-dependent access procedure. For 802.11p communication in platooning situations at junctions, the authors created models. [3] consider continual backoff freezing and use the PSFFA to describe the gearbox queue's dynamic behaviour. The access process with its four ACs, also use a z-domain linear model was demonstrated by [3]. *Figs 4.a-4.c* and *Figs 5.a-5.c*, respectively, show the time-dependent packet transmission delay and packet delivery ratio of four Acs.



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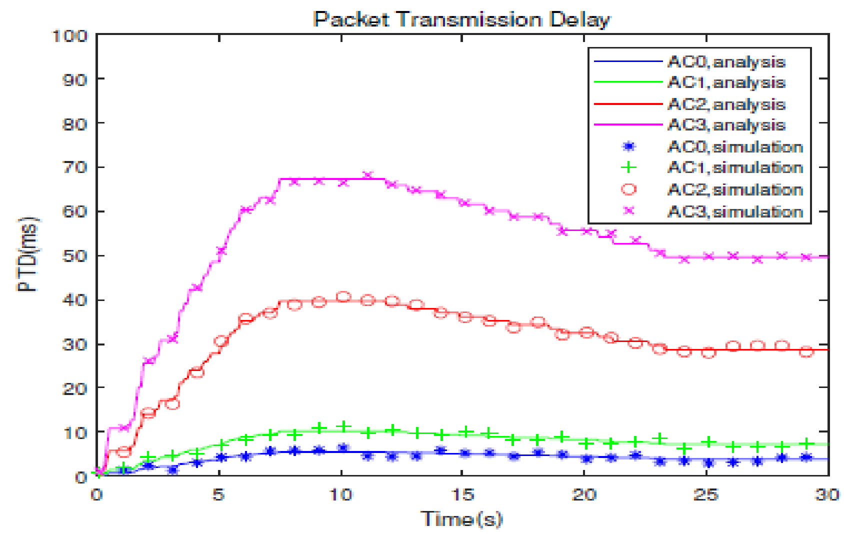
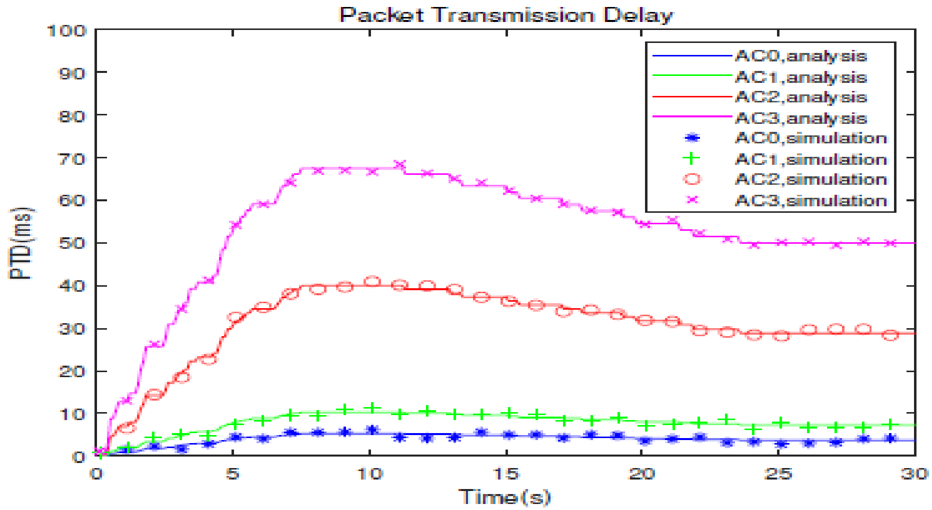
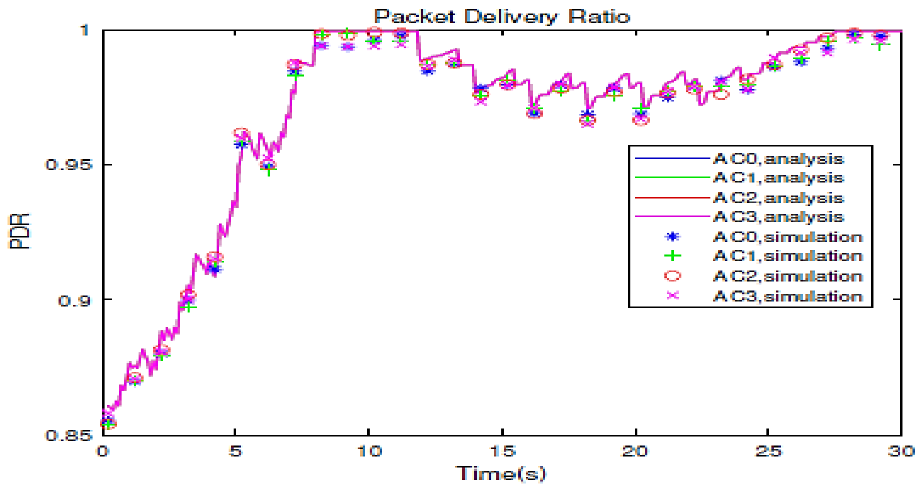


b.

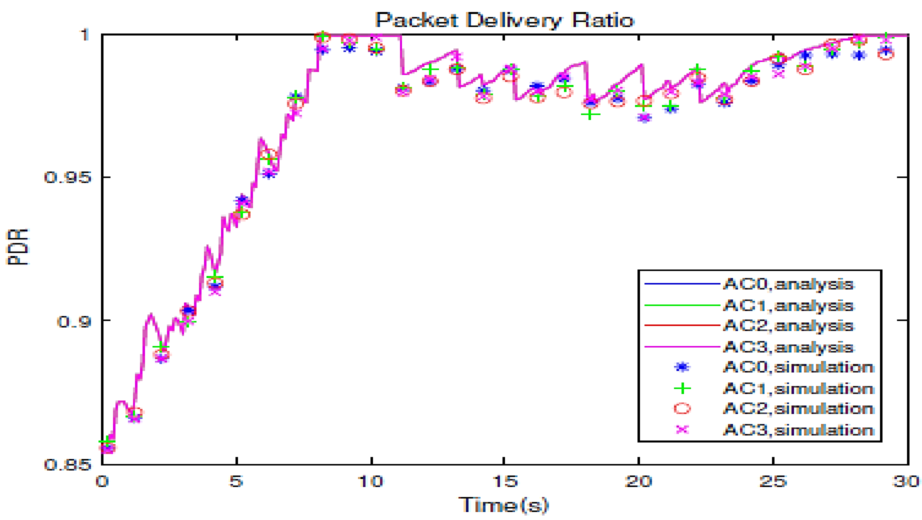


c.

Fig. 4. The packet transmission delay that is time dependent. A left turn, a straight shot, or a right turn, respectively [3].



a.



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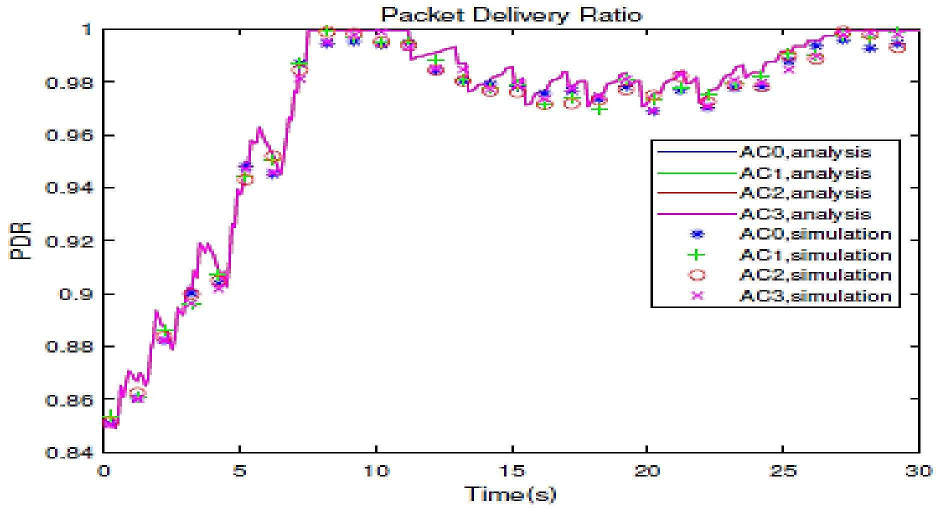

c.

Fig. 5. The proportion of time-dependent packet deliveries [3].

The findings of a study [4] into the variables influencing queue utilization dynamics on routers in telecommunication networks. The investigation shows that the average queue length reaches a steady state value following a transient process lasting from a few to tens of seconds when utilizing a dynamic model, specifically PSFFA. It is advised to calculate the average queue length using steady state estimations only after the transient process has finished and a more precise differential model may be used.

To accurately predict the average queue length while analysing the average queue length and Quality of Service [4] in a network, a dynamic model with a nonlinear differential equation must be used. Only when the transient process has ended, and the length of the transient process is controlled by variables like flow rate, router interface capacity, and service discipline, are steady-state estimations useful for determining the average queue length. A better choice of queuing models and smaller packet sizes can further hasten the average queue length's convergence

5 | Closing Remarks With Next Phase of Research

More specifically, the state variable of the underlying queue is found in this paper, which addresses a difficult problem in queueing theory. To formulate the non-stationary $M/D/1$ queueing system, the article provides a solution to this problem by using a PSFFA method. Applications of non-stationary queues in various scientific disciplines and open research problems will be the focus of future effort. Together with queueing parameters, the study also looks at how time affects the stability dynamics of the underlying queue. More fundamentally, some interesting PSFFA applications to IoT are provided. Future work involves further investigation of the impact of $\sigma(t), 1 > \sigma(t) > 0$ on the stability of $G/M/1$ PSFFA theory.

Author Contributions

Ismail A. Mageed conducted the research, developed the analytic modeling approach, and performed the numerical analysis of the Pointwise Stationary Fluid Flow Approximation (PSFFA) for the non-stationary M/D/1 queueing system. The author also applied the findings to the Internet of Things (IoT) context and wrote the manuscript.

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Data Availability

The data used in this research is derived from theoretical models and numerical analyses. All data and results are detailed in the manuscript.

Conflicts of Interest

The author declares no conflicts of interest in relation to this study.

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