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On Analysis of Oscillations of a Multilayer Structures

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Abstract

In this paper, we introduce an approach to increase sound insulation by using a multilayer building construction. As an example of such analysis, we consider sound processing of multilayer enclosing and load-bearing constructions to increase sound insulation. We also introduce an analytical approach for the analysis of the considered processes.

Keywords: Sound-insulation of building constructions, Analytical approach for prognosis.

1 | Introduction

High rates of development of construction equipment create the necessary prerequisites for the design and construction of buildings and other structures of elements that have significant strength and stability with low weight and small thickness [1–8]. At the same time, the development of technology leads to the emergence of more powerful machines and to an increasing number of vehicles, which leads to an increase in noise in populated areas and civil and industrial buildings. Acoustic improvement of premises becomes an actual problem in each design and construction of each building [9–13]. In the framework of solving this problem, the problem of the sound- insulating ability of the enclosing and supporting structures is first of all solved. To solve the problem, it is necessary to analyze the sound effect on the structure. In the framework of this paper, we analyzed sound- insulating enclosing and load-bearing building constructions. We will analyze this effect using the example of transverse oscillations of a multilayer construction under the influence of a plane sound wave perpendicular to the interface between the layers of the plate construction. The qualitative structure of the considered construction is presented in *Fig. 1*.

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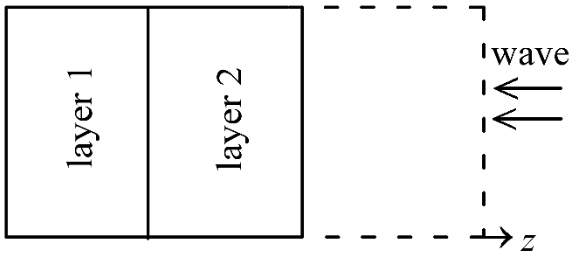

Fig. 1. Structure of the considered construction.

2 | Method of Solution

The solution of the following wave equation has determined oscillations in the considered multilayer construction.

$$\frac{\partial^2 u(x,y,z,t)}{\partial t^2}=F(x,y,z,t)+\frac{E(z)}{\rho(z)[1-\sigma(z)]}\frac{\partial^2 u(x,y,z,t)}{\partial x^2}+\frac{E(z)}{\rho(z)[1-\sigma(z)]}\frac{\partial^2 u(x,y,z,t)}{\sigma y^2}+$$

$$+\frac{\partial}{\partial z}\left[\frac{E(z)}{\rho(z)[1-\sigma(z)]}\frac{\partial u(x,y,z,t)}{\partial z}\right], \quad (3)$$

where E (z) is the modulus of elasticity; ρ (z) is the density of materials of the considered structure; σ (z) is the Poisson ratio, u (x,y,z,t) is the displacement of the points of the structure during oscillations; F (x,y,z,t) is the external processing (knock, sound wave, etc.); Lx, Ly and Lz are the dimensions of the considered structure in the directions indicated in the indices; x, y and z are spatial coordinates; t is the current time. Let us consider the case when the edges of the structure are rigidly fixed, and there is no effect on it at the time of the beginning of the considered processing. Then the boundary and initial conditions for equation (1) could be written in the following form.

$$\left.\frac{\partial u(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial u(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial u(x,y,z,t)}{\partial y}\right|_{y=0}=0,$$

$$\left.\frac{\partial u(x,y,z,t)}{\partial y}\right|_{x=L_y}=0,\ u(x,y,z,0)=0. \quad (2)$$

Now, let us solve *Eq. (1)* using the method of averaging functional corrections [14]. In the framework of the method to obtain the first-order approximation of the desired function u (x,y,z,t) one shall replace on the not yet known average value α1 in the right-hand side of the *Eq. (1)*. Then the equation for the first-order approximation of the function u (x,y,z,t) takes the form.

$$\frac{\partial^2 u_1(x,y,z,t)}{\partial t^2}=F(x,y,z,t). \quad (3)$$

Integration of both sides of *Eq. (3)* in time leads to the following result:

$$u_1(x,y,z,t)=\int_0^t (t-\tau)F(x,y,z,\tau)d\tau. \quad (3.a)$$

The not yet known average value α1 of the function u (x,y,z,t) should be determined using the standard relation.

$$\alpha_1 = \frac{1}{L_x L_y L_z \Theta}\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} u_1(x,y,z,t)\,dz\,dy\,dz\,dt, \tag{4}$$

where Θ is the continuance of observation of the oscillation in the considered structure. Substitution of the *Relation (3.a)* into *Relation (4)* leads to the following result:

$$\alpha_1 = -\frac{1}{2L_x L_y L_z \Theta}\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} t^2 F(x,y,z,\tau)\,dz\,dy\,dz\,dt. \tag{4.a}$$

The second-order approximation of the function u (x,y,z,t) was calculated by replacing of the considered function in the right-hand side of the *Eq. (1)* on the sum of the approximation of the previous order and the average value of the considered approximation α2, i.e. by the sum α2+u1(x,y,z,t). Then the equation for the second-order approximation of the function u (x,y,z,t) takes the form

$$\frac{\partial^2 u_2(x,y,z,t)}{\partial t^2} = F(x,y,z,t) + \frac{E(z)}{\rho(z)\left[1-\sigma(z)\right]}\frac{\partial^2 u_1(x,y,z,t)}{x^2} + \frac{E(z)}{\rho(z)\left[1-\sigma(z)\right]}\frac{\partial^2 u_1(x,y,z,t)}{\partial y^2} + \frac{\partial}{\partial z}\left[\frac{E(z)}{\rho(z)\left[1-\sigma(z)\right]}\frac{\partial u_1(x,y,z,t)}{\partial z}\right]. \tag{5}$$

The solution of *Eq. (5)* could be written as

$$u_2(x,y,z,t) = \frac{E(z)}{\rho(z)\left[1-\sigma(z)\right]}\int_0^t (t-\tau)\frac{\partial^2 u_1(x,y,z,\tau)}{\partial x^2}d\tau + \frac{E(z)}{\rho(z)\left[1-\sigma(z)\right]}\int_0^t (t-\tau)\frac{\partial^2 u_1(x,y,z,\tau)}{\partial y^2}d\tau + \frac{\partial}{\partial z}\int_0^t \frac{E(z)}{\rho(z)\left[1-\sigma(z)\right]}\frac{\partial u_1(x,y,z,\tau)}{\partial z}d\tau + \int_0^t (t-\tau)F(x,y,z,\tau)\,d\tau. \tag{5.a}$$

The average value α2 of the second-order approximation of the considered function u (x,y,z,t) should be calculated by using the following standard relation [14].

$$\alpha_n = \frac{1}{L_x L_y L_z \Theta}\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[u_n(x,y,z,t) - u_{n-1}(x,y,z,t)\right]dz\,dy\,dz\,dt, \tag{6}$$

where n is the order of the required approximation, substitution of *Relation (3.a)* and *Relation (5.a)* into *Relation (6)* leads to the following result.

$$\alpha_2 = -\frac{1}{2L_x L_y L_z \Theta}\int_0^{\Theta} t\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\frac{E(z)}{\rho(z)\left[1-\sigma(z)\right]}\frac{\partial^2 F(x,y,z,t)}{\partial x^2}dz\,dy\,dz\,dt - \frac{1}{2L_x L_y L_z \Theta}\int_0^{\Theta} t\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\frac{E(z)}{\rho(z)\left[1-\sigma(z)\right]}\frac{\partial^2 F(x,y,z,\tau)}{\partial y^2}dz\,dy\,dz\,dt - \frac{1}{2L_x L_y L_z \Theta}\int_0^{\Theta} t\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\frac{\partial}{\partial z}\left[\frac{E(z)}{\rho(z)\left[1-\sigma(z)\right]}\frac{\partial F(x,y,z,\tau)}{\partial z}\right]dz\,dy\,dz\,dt. \tag{6.a}$$

Analysis of the spatio-temporal distribution of the displacement of points of the considered structures during oscillation has been done analytically in the framework of the second-order approximation by the method of averaging the functional corrections. The approximation is usually enough for qualitative analysis and obtaining some quantitative results. The results of the analytical calculations were verified by comparison with the results of the direct numerical simulations.

3 | Discussion

In this section we analyzed the spatio-temporal distribution of the displacement of the points of the considered multilayer building construction during their oscillations under the influence of a plane wave F (x,y,z,t) = A·exp(ω t-kzz), where A is the amplitude of the wave, kz is the projection of the wave number on the Oz axis, and ω is the wave frequency. *Fig. 1* shows the qualitative spatial distribution of the displacement of points of the considered structure as a function of the coordinates x and y at a fixed value of time. *Fig. 2* shows the qualitative spatio-temporal distribution of the displacement of the considered structure points as a function of the coordinate z and time t for fixed values of the x and y coordinates.

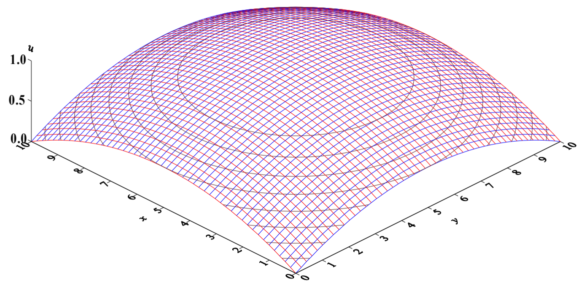

Fig. 2. The qualitative spatial distribution of the displacement of the points of the considered structure as a function of the coordinates x and y at a fixed time.

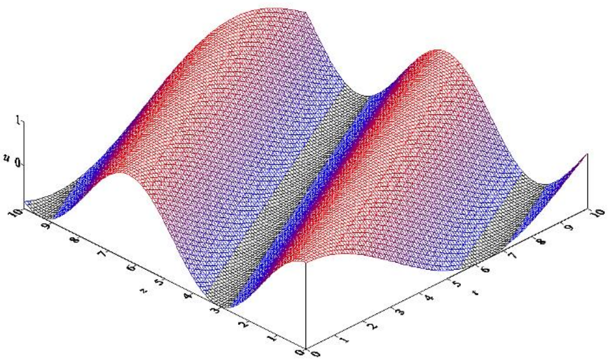

Fig. 3. The qualitative spatio-temporal distribution of the displacement of points of the considered structure as a function of the coordinate z and time t for fixed values of the coordinates x and y.

Analysis of the spatio-temporal distribution of the considered displacement shows, that compromise between increasing of sound-insulation of by multilayer constructions and increasing of their complications are using two layer structures, which are presented on *Fig. 4*. These structures consists of load-bearing layer for reflection of received signal (see layer 1 on *Fig. 4*) and facing layer for absorption of the received signal (see layer 2 on *Fig. 4*).

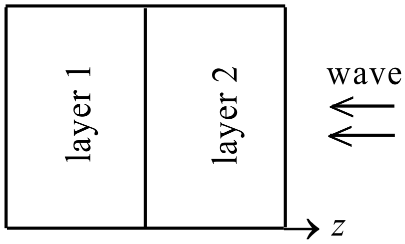

Fig. 4. Optimal multilayer structure for sound insulation.

4 | Conclusion

In this paper, we introduce a model for the prognosis of sound insulation by a myltilayer building construction. To analyze the model, we introduce an analytical approach for the analysis of oscillations of points of the considered structure during sound processing. We formulate recommendations to increase the sound insulation of the considered constructions.

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