Mechanical Technology and Engineering Insights

www.mtei.reapress.com

Research Expansion Alliance

Mech. Technol. Eng. Insights . Vol. 1, No. 4 (2024) 198–208.

Paper Type: Original Article

Influence of Optical Thickness and Inclination Angle on Coupled Natural Convection and Thermal Radiation in Inclined Cavities

Khadijeh Ghaziani[1,*], Omar Mar Cornelio[2]

[1] Department of, Ayandegan University, Tonekabon, Iran; ghaziyani89@gmail.com.

[2] Centro De Estudio De Matemática Computacional, Universidad De Las Ciencias Informáticas, 19370 La Habana, Cuba; omarmar@uci.cu.

Citation:

Received: 19 May 2023 *Revised: 18 September 2023* *Accepted: 08 December 2023*	Ghaziani, K., & Cornelio, O. M. (2024). Influence of optical thickness and inclination angle on coupled natural convection and thermal radiation in inclined cavities. *Mechanical Technology and Engineering Insights, 1*(4), 198-208.

Abstract

In this study, the effect of thermal radiation on heat transfer via natural convection in inclined cavities under inclination angles of 45° and 60° is studied numerically. For modeling the flow field and heat transfer, the conservation laws of mass, momentum, and energy are solved using the Finite Volume Method (FVM). In contrast, the SIMPLE algorithm is employed to solve the pressure-velocity coupling. The buoyancy force resulting from the temperature gradient is considered using the Boussinesq approximation. In the first step, the characteristics of natural convection heat transfer in inclined cavities are studied in the absence of radiation. Then, the effect of thermal radiation is studied for radiative participation and non-participation of the fluid medium. To assess the accuracy of numerical solutions, results such as streamlines, isotherms, and Nusselt number profiles are compared with those reported in the literature and found to be in close agreement. The results show that the optical thickness has a considerable effect on the flow field, the strength of recirculation, the temperature profile, and, ultimately, the heat transfer performance. In other words, the importance of radiation heat transfer increases with increasing optical thickness. Consequently, the average Nusselt number decreases as the optical thickness increases. Moreover, an increase in the angle of inclination weakens buoyancy-induced flow, thereby reducing recirculation and heat transfer. The combined influence of geometric inclination and radiation reveals a significant interaction between fluid mechanics and heat transfer, which is crucial to understanding the thermo-fluid dynamics of inclined cavities. The results obtained from the present investigation are expected to provide valuable information for the design of optimal thermal systems in which interactions between convection and radiation play a key role.

Keywords: Thermal radiation heat transfer, Natural convection, Optical thickness, Inclined cavity, Finite volume method, Nusselt number.

1 | Introduction

Natural convection is considered one of the basic modes of heat transfer in technical systems. It is used in equipment such as solar collectors, thermal energy storage systems, electronic cooling systems, heat exchangers, and nuclear reactors. The widespread use of this phenomenon has generated great interest in

Corresponding Author: ghaziyani89@gmail.com

https://doi.org/10.48313/mtei.v1i4.67

researching natural convection, resulting in numerous papers published on this topic in recent decades. Nevertheless, in classical research on natural convection, the radiative effect was usually disregarded because it is applicable only at moderate temperatures. However, in high-temperature systems, this assumption is invalid, as radiation heat transfer plays a significant role and can be on par with other processes.

In the case of forced convection, it is quite simple to consider convection and radiation independently and sum their effects. Nonetheless, this Approach cannot be used for natural convection because it is interrelated with other parameters. Namely, the temperature distribution changes the velocity distribution through the buoyancy effect, whereas radiation influences the temperature distribution. Thus, there is a strong coupling among the flow field, the thermal field, and radiative heat transfer. Hence, an accurate analysis requires solving the Navier-Stokes equations, energy equations, and the Radiative Transfer Equation (RTE) simultaneously.

Problems in radiative heat transfer can be classified into two types. The first type concerns a radiatively non-participating medium in which radiation interacts only at surfaces and can be analyzed using view factors and surface energy balance relations. In the other type, the medium is radiatively participating, meaning absorption, emission, and scattering occur within it. Here, the radiation field is analyzed based on the RTE, an integro-differential equation. In this study, both the medium's radiation effects and convection-radiation interactions are considered to obtain more accurate results.

Enclosures subject to natural convection have been widely used as standard test cases to validate CFD methods. Extensive research has been conducted on square and rectangular cavities due to their simple geometry and the availability of benchmark data [1–3]. However, studies on the interaction between natural convection and radiation have not been adequately conducted.

The earliest studies were focused on the interaction between natural convection and radiation inside enclosures. Yücel et al. [4] used the streamfunction-vorticity Approach, along with a product integration technique, to analyze the two-dimensional square cavity filled with a participating radiative medium. The influence of inclination of the cavity on natural convection-radiation interactions in vertical and inclined square enclosures was studied by Bouali et al. [5], revealing that the orientation of the cavity plays a significant role in the flow and heat transfer characteristics. Natural convection coupled with gas radiation within an enclosure was experimentally and numerically analyzed by Yücel et al. [4], indicating the need for radiative participation in high-temperature systems. The analysis was further extended to 3D cavities by Hasani [6] in participating and non-participating cases. Hernández-Castillo et al. [7] further explored the radiative effect on three-dimensional cavities and found it to be considerable for the thermal field.

In addition to the above, other researchers have focused on real-life problems involving interactions between convection and radiation. For instance, Shulepova et al. [8] explored the effects of mixed convection and radiation on lid-driven cavities. In another study, Mahapatra et al. [9], Mikhailenko et al. [10] examined the effect of radiation on mixed convection in inclined and vertical cavities and found that thermal radiation reduces the intensity of the buoyant force but increases local heat transfer characteristics, such as the Nusselt number and wall friction factor. Mikhailenko et al. [11], Al-Hafidh et al. [12] further explored radiation effects in rectangular cavities with varying optical thicknesses, conduction-radiation parameters, refractive indices, and wall scattering.

Recently, it was shown that radiation plays an important role in influencing fluid flow, temperature field, entropy generation, and heat transfer efficiency within enclosures. Although many investigations have already been conducted in this area, most studies focus on simpler geometric shapes, such as squares or rectangles. On the other hand, although inclined cavities have been considered significant from a practical point of view, few investigations have been performed on such enclosures. Recent review studies have emphasized that optical thickness, radiation–conduction coupling, and geometric orientation play a critical role in determining thermo-fluid behavior in enclosed systems [5], [13]. Therefore, there is a clear need for further investigations into complex geometries, such as inclined cavities, under the coupled effects of natural convection and thermal radiation.

Motivated by these considerations, the present study numerically investigates natural convection heat transfer in inclined cavities with thermal radiation. The effects of optical thickness and cavity inclination angle on flow structure, temperature distribution, and heat transfer characteristics are examined in detail.

2 | Problem Formulation and Assumptions

The following assumptions are adopted in the present numerical study:

I. The flow is assumed to be laminar, Newtonian, incompressible, and two-dimensional.

II. Thermophysical properties of the fluid are considered constant.

III. The Boussinesq approximation is employed to model buoyancy effects induced by temperature variations.

IV. The viscous dissipation term in the energy equation is neglected.

V. The participating medium is assumed to be gray with respect to thermal radiation.

VI. The radiative heat transfer within the medium is considered significant; hence, the medium is treated as radiatively participating.

VII. The refractive index of both the medium and the boundaries is assumed to be constant and equal to unity.

Based on the above assumptions, the governing equations for mass, momentum, and energy conservation in Cartesian coordinates can be expressed as follows:

$$\frac{\partial u_i}{\partial x_i} = 0. \quad (1)$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial xi} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \beta_T (T - T_{ref})\delta_{i2}. \quad (2)$$

$$\frac{\partial T}{\partial t} + \frac{\partial u_i T}{\partial x_j} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial x_j \partial x_j} - \frac{1}{\rho C_p}\nabla . q_R, \quad (3)$$

where $\nabla . q_R$ represents the radiative heat flux vector, which is defined as:

$$\nabla . q_R = \kappa(4\pi I_b - G). \quad (4)$$

In the above relation, (κ) is the absorption coefficient, (I_b) is the Stefan–Boltzmann constant, (T_b) is the blackbody temperature, and (G) denotes the incident radiation, which is obtained by integrating the radiation intensity over all directions:

$$G = \int_{4\pi} I d\lambda. \quad (5)$$

The radiation intensity is determined by solving the RTE, which is expressed as:

$$\xi_i \frac{\partial I}{\partial x_i} + \beta I = \kappa I_b + \frac{\sigma_s}{4\pi}\int_{4\pi} I\Phi(\xi i, \xi_i')d\Omega', \quad (6)$$

where ($\xi_i (i = 1,2,3)$) denotes the direction vector along which theintensity (I) is evaluated. The refractive index (σ_s), β extinction coefficient, and $\Phi(\xi_i, \xi_i')$ scattering phase function are assumed to be constant in this study.

The total heat flux at the wall consists of both convective and radiative components and can be written as:

$$q_{tw} = q_{Cw+}q_{Rw}. \quad (7)$$

In this relation, q_{Cw} and q_{Rw} represent the convection heat flux and the radiation heat flux, respectively, and are calculated from the following relations:

$$q_{Cw} = -k\frac{\partial T}{\partial n}. \quad (8)$$

$$q_{Rw} = \int (\hat{n}.s)|_{w} D\omega. \quad (9)$$

The local Nusselt number at the wall is defined based on both convective and radiative contributions as:

$$Nu = Nu_C + Nu_R. \quad (10)$$

$$Nu_C = \frac{q_{Cw} L}{k(T_h - T_c)}. \quad (11)$$

$$Nu_R = \frac{q_{Rw} L}{k(T_h - T_c)}. \quad (12)$$

The average Nusselt number is calculated by integrating the local Nusselt number over the heated wall surface.

3 | Numerical Approach

The mathematical expressions representing mass, momentum, energy, and radiative heat transfer were solved numerically using a finite-volume technique. The computational grid consisted of quadrilateral control volumes that partitioned the physical domain into small segments. Integral forms of the governing equations were considered in each control volume to satisfy the conservation laws of mass, momentum, and energy in the physical domain. Pressure-velocity coupling was accounted for by the Semi-Implicit Method for Pressure-Linked Equations (SIMPLE) algorithm. At each time step, the momentum equations were first solved for an intermediate velocity field. Then, the pressure-correction equation, derived from the continuity equation, was solved, and the pressure and velocity fields were updated accordingly. The central differencing Approach numerically solved the diffusion term, while the convection terms were solved by an upwind differencing scheme to guarantee accuracy and stability. Radiative heat transfer within the participating medium was accounted for by solving the radiative heat transfer equation. The divergence of the radiative heat flux was computed and included as a source term in the energy equation. Hence, both radiation and natural convection were considered in the energy equation. In each iteration, the radiative source term was determined from the temperature distribution, and the system was solved again until convergence was achieved.

Numerical solution of the problem involved iteratively solving the momentum equations, the pressure-correction equation, the RTE, and the energy equation until the solution satisfied all governing equations with the required convergence criteria. Numerical convergence was judged based on the normalized residuals of the governing equations, such that calculations were considered to have converged once the residuals of the continuity and momentum equations were below (10^{-3}).

In contrast, the residual of the energy equation was lower than (10^{-6}) To verify the independence of the numerical solution with respect to grid spacing, a series of sensitivity studies was conducted using three different mesh densities: 60 × 60, 80 × 80, and 100 × 100 control volumes. The average Nusselt number at the heated wall surface was selected as a monitoring quantity for sensitivity studies. It was observed that there was a negligible deviation between the results obtained by 80 × 80 and 100 × 100 meshes. Hence, an 80 × 80 mesh was considered to be adequate for future analyses.

4 | Results and Discussion

Fig. 1 shows the geometry used in this study and its corresponding boundary conditions. The cavity used in this analysis has a unity aspect ratio, where the inclined sides incline (γ) degrees relative to the horizontal plane. In this analysis, two inclination angles were used, namely ($45^{\circ} = \gamma$) and ($60^{\circ} = \gamma$).

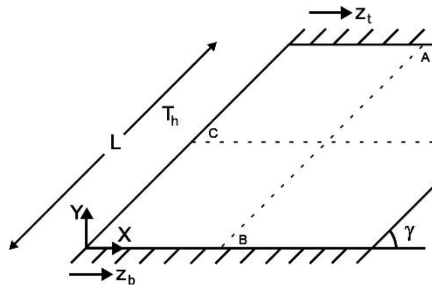

Fig. 1. Geometry and boundary conditions of the inclined cavity.

The top and bottom walls are considered adiabatic, while the inclined walls are maintained at constant but different temperatures. The temperature ratio between the hot and cold walls is set to 2, with the cold wall temperature fixed at 500 K. The cavity is filled with a Newtonian fluid having a Prandtl number of 0.71. The Rayleigh number is kept constant at (10^5), along with the wall temperature ratio and cold wall temperature, in order to isolate the effect of optical thickness. The problem is solved for different values of optical thickness ($\tau = 0, 0.5, 1.0, 10.0$) as well as for the pure natural-convection case. It is important to note that the pure convection case differs from the radiative non-participating medium case. In the former, radiative heat exchange is completely neglected, whereas in the latter, surface-to-surface radiation is still allowed but without participating media effects. In the present study, the refractive index is assumed to be unity, and the emissivity of the walls is set to 1.

To validate the numerical model, the present results are compared with the benchmark solution of Kumar and Suresh [15] in terms of the dimensionless temperature distribution along line C–D for a 60° inclined cavity at (Ra = 10^5). As shown in *Fig. 2*, an excellent agreement is observed between the present results and the reference data, confirming the accuracy of the numerical formulation.

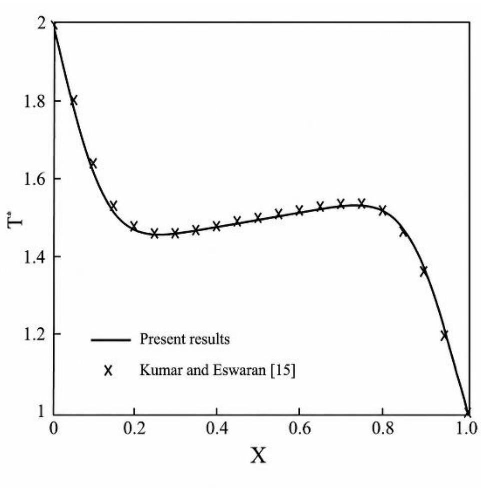

Fig. 2. Comparison of dimensionless temperature distribution along line C–D for a 60° inclined cavity at 10^5).

Fig. 3 presents the streamlines and isotherm contours for the 60° inclined cavity at (10^5) under different optical thicknesses. The results clearly indicate that the optical thickness significantly influences both the flow structure and the temperature distribution within the cavity. At lower optical thicknesses, radiative heat transfer is more effective at redistributing thermal energy, leading to stronger thermal gradients and more pronounced circulation patterns. As the optical thickness increases, the medium becomes more optically dense, reducing the penetration of thermal radiation and weakening its contribution to overall heat transfer. Consequently, the flow intensity and thermal stratification are notably affected.

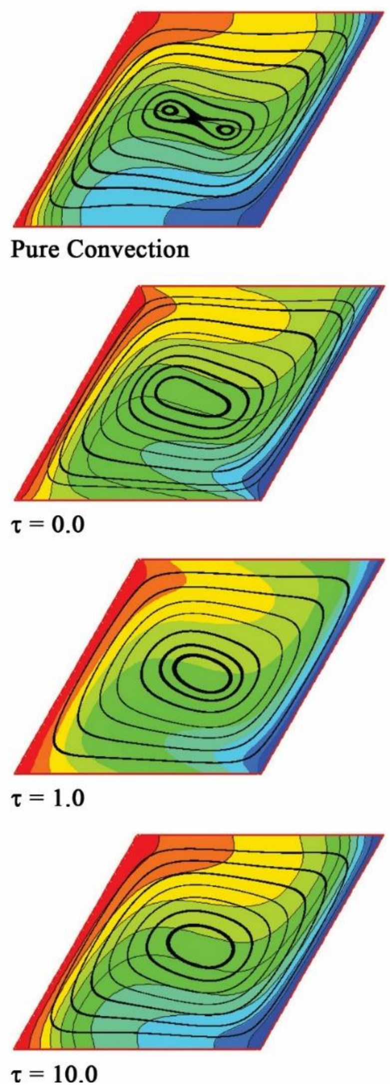

Fig. 3. Streamlines and isotherm contours for a 60° inclined cavity at (10^5) for different optical thicknesses.

Fig. 3 presents the streamlines and isotherm contours for a 60° inclined cavity at a Rayleigh number of 10^5, comparing optical-thickness values with the pure natural-convection case. The results clearly indicate that the inclusion of thermal radiation leads to a noticeable alteration in the flow structure. This behavior is also observed for the temperature field.

The characteristic two-cell flow structure observed in the pure natural-convection case disappears once radiative heat transfer is introduced, and a single dominant circulation cell forms for all considered optical-thickness values. The central streamlines are elliptical, and at lower optical thicknesses, the vortex core is elongated along the cavity's larger dimension. However, as the optical thickness increases, the flow structure becomes more symmetric and gradually approaches a circular pattern. This trend indicates that higher optical thickness reduces radiative penetration, leading to a more uniform thermal field and, consequently, a more stable and symmetric flow pattern.

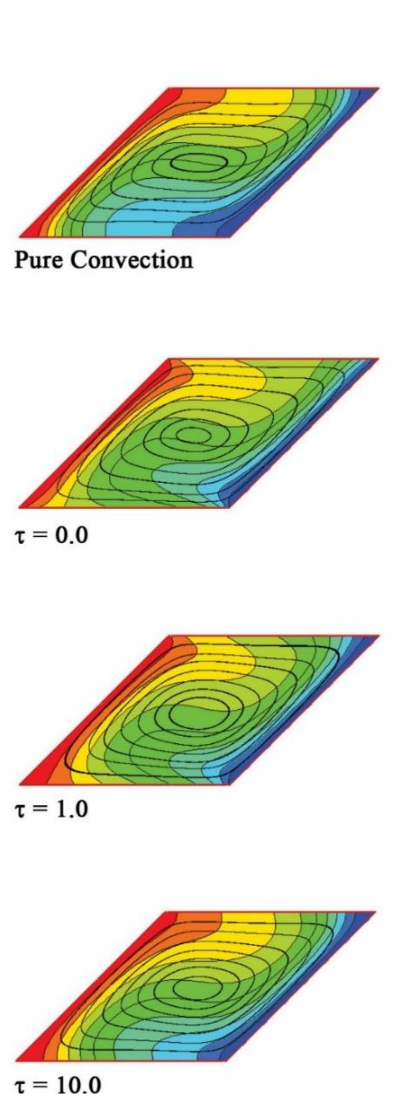

Fig. 4. Streamlines and isotherms for a Rayleigh number of 10^5 and a cavity inclined at 45° degrees for various optical thicknesses.

For a radiatively transparent medium ($\tau=0$), streamlines exhibit characteristics distinct from those of other radiatively participating media. In this case, the central core is not perfectly elliptical and is also offset relative to the longer diagonal of the cavity. Isotherms shown in *Fig. 3* are symmetric about the cavity center under pure convection conditions. However, when radiation is considered, they become more concentrated near the cold wall. This is because, for a radiatively participating medium, the region near the hot wall is more intensely heated by radiation. This heated region expands as the optical thickness increases. For a radiatively transparent medium ($\tau=0$), the temperature gradient is steep near the adiabatic walls, since the medium absorbs no heat and the entire radiative flux reaches them. In contrast, for nonzero optical thickness values, the medium absorbs a portion of the thermal energy, thereby reducing the temperature gradient. As the optical thickness increases, the curvature of the isotherms decreases, indicating an enhancement in thermal diffusion effects due to radiation.

Fig. 4 illustrates the streamlines and isotherms for an inclined cavity at 45 degrees with a Rayleigh number of 10^5 for different optical thicknesses alongside the pure convection case. Results show that when radiative heat transfer is considered, the central streamlines become elliptical and oriented toward the longer diagonal of the cavity. The contours of streamlines and isotherms exhibit behavior similar to that of the 60 cavities.

It is interesting to observe the balance between radiative and convective heat transfer along the adiabatic walls. *Figs. 5* and *6* present the convective and radiative Nusselt numbers on the bottom and top walls, respectively. From *Fig. 5*, it is evident that the bottom wall is cooled (Nu<0) by convection for a short distance from the left corner and subsequently heated (Nu>0). In contrast, it is first heated and then cooled by radiation. This trend is reversed for the top wall (*Fig. 6*). Clearly, the sum of the two Nusselt numbers (radiative and convective) equals zero, as the walls are considered adiabatic. The magnitudes of heating and cooling decrease with increasing optical thickness. The magnitude of heating (or cooling) on the bottom wall exceeds that on the top wall.

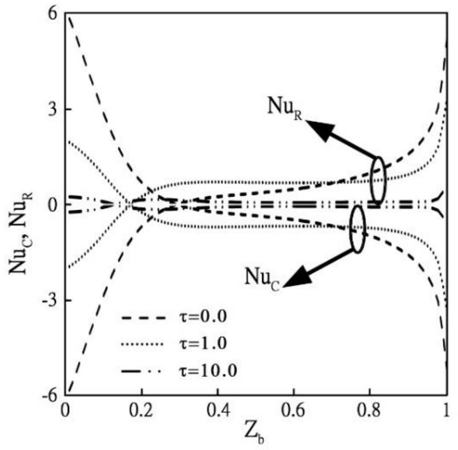

Fig. 5. Variation of Nusselt numbers on the bottom wall for an inclination angle of 60° and a Rayleigh number of 10^5.

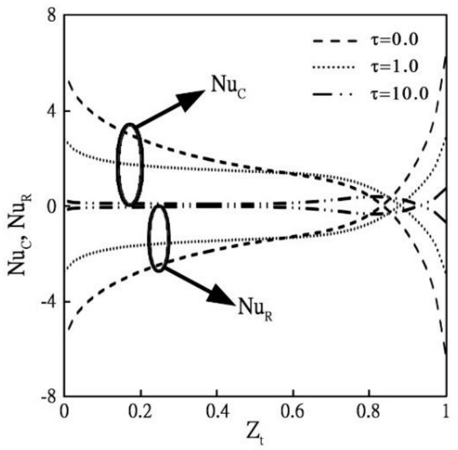

Fig. 6.Variation of Nusselt numbers on the top wall for an inclination angle of 60° and a Rayleigh number of 10^5.

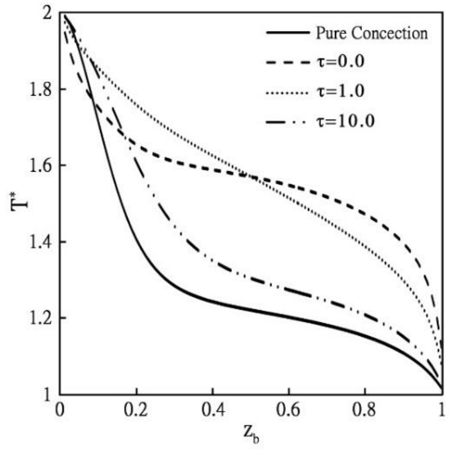

Fig. 7. Temperature variation on the bottom wall for an inclination angle of 60° and a Rayleigh number of 10^5.

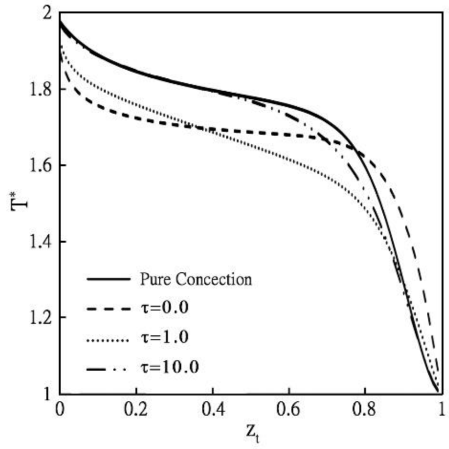

Fig. 8. Temperature variation on the top wall of the cavity with an inclination angle of 60° and a Rayleigh number of 10^5.

Temperature distributions on the bottom and top adiabatic walls are presented in *Figs. 7* and *8*, respectively. Under pure convection conditions, a large portion of the bottom wall is cooled, while a large portion of the top wall is heated. Radiation increases the temperature of the bottom wall and decreases the temperature of the top wall. The magnitudes of heating and cooling on the bottom and top walls diminish as the optical thickness increases.

4 | Conclusion

In the current study, the combined effects of natural convection and thermal radiation in inclined square enclosures were investigated comprehensively. Inclination angles of 45° and 60° were used for both radiative and non-radiative enclosure types. Key observations can be made as follows:

I. Modification of flow and thermal fields: Streamlines and isotherms are strongly affected by thermal radiation. Compared with natural convection alone, the addition of radiation changes the core flow pattern: the central streamlines adopt elliptical trajectories, running parallel to the enclosure's longer diagonal. Isotherms also move closer to the cold surface of the enclosure.

II. Radiation as the primary heat transfer mechanism: At a temperature difference of 2 K between the hot and cold surfaces, with the cold surface held at 500 K, thermal radiation becomes the dominant mode of heat transfer. In this case, not only does the thermal energy distribution increase, but the convective flow intensity does as well.

III. Thermal effects of opposite nature on top and bottom walls due to radiation: Radiation causes a cooling effect on the top adiabatic wall, while the bottom adiabatic wall experiences heating due to radiation. This thermal effect is independent of inclination angles. It is caused by unidirectional energy exchange between the hot and cold surfaces and the participating fluid.

IV. Optical thickness effect on the total heat transfer coefficient: The total Nusselt number associated with the walls declines with increasing optical thickness. Such a decline in the Nusselt number is caused by high absorption and emission of radiant energy, resulting in lower temperature gradients and, hence, lower Nusselt numbers. On the other hand, when the fluid medium is radiatively transparent, high temperature gradients and therefore Nusselt numbers are experienced.

Conclusively, thermal radiation significantly affects both the thermal and flow characteristics of the fluid flowing through an inclined square cavity, especially when the fluid is radiatively participating, and the temperature ratio is relatively high.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability

All data are included in the text.

Funding

This research received no specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

References

[1] Bergman, T. L. (2011). *Fundamentals of heat and mass transfer*. John Wiley & Sons. https://books.google.com/books/about/Fundamentals_of_Heat_and_Mass_Transfer.html?id=0kAxEAAAQBAJ

[2] Cuevas, J. C., & García-Vidal, F. J. (2018). Radiative heat transfer. *Acs photonics*, *5*(10), 3896–3915. http://webs.ftmc.uam.es/plasmonanoquanta.group/pdfs/149.pdf

[3] Bejan, A. (2013). *Convection heat transfer*. John wiley & sons. https://www.amazon.com/s?k=9780470900376&i=stripbooks&linkCode=qs

[4] Yücel, A., Acharya, S., & Williams, M. L. (1989). Natural convection and radiation in a square enclosure. *Numerical heat transfer, part a: Applications*, *15*(2), 261–278. https://doi.org/10.1080/10407788908944688

[5] Bouali, H., Mezrhab, A., Amaoui, H., & Bouzidi, M. (2006). Radiation—natural convection heat transfer in an inclined rectangular enclosure. *International journal of thermal sciences*, *45*(6), 553–566. https://doi.org/10.1016/j.ijthermalsci.2005.10.001

[6] Hasani, S. M. F. (1998). *Combined natural convection and radiation in a triangular enclosure*. The University of Akron. https://search.proquest.com/openview/5cf9a673d6f33f87018e0f4a494f1803/1?pq-origsite=gscholar&cbl=18750&diss=y

[7] Hernández-Castillo, P., Castillo, J. A., & Huelsz, G. (2022). Heat transfer by natural convection and radiation in three dimensional differentially heated tall cavities. *Case studies in thermal engineering*, *40*, 102529. https://doi.org/10.1016/j.csite.2022.102529

[8] Shulepova, E. V, Sheremet, M. A., Oztop, H. F., & Abu-Hamdeh, N. (2021). Mixed convection–radiation in lid-driven cavities with nanofluids and time-dependent heat-generating body. *Journal of thermal analysis and calorimetry*, *146*(2), 725–738. https://doi.org/10.1007/s10973-020-10003-7

[9] Mahapatra, T. R., Pal, D., & Mondal, S. (2013). Mixed convection flow in an inclined enclosure under magnetic field with thermal radiation and heat generation. *International communications in heat and mass transfer*, *41*, 47–56. https://doi.org/10.1016/j.icheatmasstransfer.2012.10.028

[10] Mikhailenko, S. A., Miroshnichenko, I. V, & Sheremet, M. A. (2021). Thermal radiation and natural convection in a large-scale enclosure heated from below: Building application. *Building simulation*, *14*(3), 681–691. https://doi.org/10.1007/s12273-020-0668-4

[11] Mandal, S. K., Deb, A., & Sen, D. (2019). Mixed convective heat transfer with surface radiation in a rectangular channel with heat sources in presence of heat spreader. *Thermal science and engineering progress*, *14*, 100423. https://doi.org/10.1016/j.tsep.2019.100423

[12] Al-Hafidh, M. H. (2008). Parametric study of mixed convective radiative heat transfer in an inclined annulus. *Al-khwarizmi engineering journal*, *4*(4), 45–56. https://alkej.uobaghdad.edu.iq/index.php/alkej/article/view/608/571

[13] Abdulkadhim, A., Abed, I. M., & Mahjoub Said, N. (2021). Review of natural convection within various shapes of enclosures. *Arabian journal for science and engineering*, *46*(12), 11543–11586. https://doi.org/10.1007/s13369-021-05952-6