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Multi-Objective Optimization of a Fuzzy Tracking Controller Using Differential Evolution for Nonlinear Three-Dimensional Tracking

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Abstract

The issue of three-dimensional tracking in a system involving nonlinear dynamics, couplings, uncertainties, and varying working conditions has become a challenge in current control engineering practice. It is of vital importance that in such a case, the precision of tracking is maximized while the control effort and energy consumed are minimized. In this paper, the optimized Fuzzy Logic Controller (FLC) has been designed for the three-dimensional tracking of a missile (satellite) through application of the Differential Evolution (DE) algorithm. The fuzzy membership function parameters are considered as design parameters, and the minimization of both the tracking error and energy is performed simultaneously as conflicting objectives. The parameters of the membership functions have been optimized through the use of the DE algorithm, and the performance of the controller has been enhanced. The results obtained through simulation prove that the fuzzy control based on DE yields an ideal compromise between tracking performance and energy utilization as compared to Proportional, Integral, and Derivative (PID) and Genetic Algorithm (GA)-based controllers in terms of overshoot and settling time. The proposed method serves as an efficient technique for the optimal design of fuzzy controllers for tracking problems in three-dimensional space.

Keywords: Fuzzy logic control, Fuzzy controller, Three-dimensional tracking, Missile tracking, Differential evolution, Multi-objective optimization, Genetic algorithm.

1 | Introduction

The control of trajectories is a central task in contemporary control engineering that is applicable in mobile robots, automatic cars, Uncrewed Air Vehicles (UAVs), aerospace systems, marine vessels, and intelligent

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transport systems. The key purpose of trajectory tracking control is for the actual state of the dynamical system to accurately track the desired trajectory with minimal tracking error while assuring stability and satisfactory performance. However, in real-life systems, the controller performance is not solely judged by the ability to perform accurate tracking. Control efforts, energy usage, transient time, disturbance rejection, and actuator constraints are other important factors to take into account. They become even more challenging when designing a control system for a nonlinear dynamical system under uncertainty, disturbances, and changing operating conditions. Thus, the creation of intelligent controllers for accurate tracking with low control efforts is a current research direction.

Traditional model-based control approaches have found extensive application in trajectory tracking due to their sound theoretical background. However, the performance of these approaches is largely dependent on the accuracy of the mathematical model and proper tuning of the controller parameters. For complex and highly nonlinear systems, developing an accurate model is not always feasible, and errors in the modeling process and unmodeled dynamics could greatly impact the performance of the controller. In addition, a controller that is developed for a particular operating point might fail if changes occur in the dynamics of the system or operating conditions.

Fuzzy Logic Control (FLC) is one of the most mature intelligent control techniques designed for nonlinear and uncertain processes. The theoretical basis of fuzzy set theory was developed by Zadeh [1], whereas the applicability of linguistic IF-THEN rules in control was shown by Mamdani and Assilian [2]. Fuzzy control systems can model relationships between control variables in terms of linguistic rules and take advantage of human expertise in designing control. Such properties make FLC a promising approach to nonlinear systems, where it is hard to build an accurate mathematical model. However, the effectiveness of fuzzy control greatly depends on the choice of parameters, such as membership functions, scaling factors, and rule-base parameters. Classical manual tuning and trial-and-error methods become impractical with the increase in the number of these parameters.

This is where evolutionary optimization techniques come into play, as they can effectively search through nonlinear parameter spaces without the need to know gradients. Among these techniques, Differential Evolution (DE), proposed by Storn and Price [3], has been widely used due to its simplicity and ability to efficiently search through continuous problem domains having a small number of control parameters. The usage and development of DE in various optimization tasks have been extensively studied [4], [5], [6]. Thus, DE seems to be applicable to optimizing continuous fuzzy membership function parameters.

Several works were dedicated to the application of fuzzy control systems in combination with evolutionary optimization algorithms. Specifically, Pishkenari et al. [7] used DE and Genetic Algorithm (GA) to optimize a fuzzy controller for trajectory tracking of the CEDRA mobile robot. They showed the superiority of these optimized membership functions in terms of performance compared to manually designed ones. Later, Pishkenari et al. [8] utilized DE for optimal design of a fuzzy trajectory-tracking controller and compared its performance with that of GA.

The ability of DE to perform optimization in fuzzy controller design has also been exhibited in nonlinear control systems. Chen et al. [9] introduced an adaptive mutation-based DE technique for TSK-type fuzzy controller optimization where membership function parameters and consequent weight parameters are optimized together. This technique was tested on several nonlinear control problems, and it showed superior search ability and lower control errors. Ochoa et al. [10] introduced a parameter adaptation DE approach to optimize the Type-1 and Interval Type-2 fuzzy controllers. Another work by Ochoa et al. [11] utilized the Interval Type-3 fuzzy system to dynamically adapt the crossover parameter of DE in order to optimize the fuzzy controller of a direct current motor.

DE has also been used in solving multi-objective fuzzy control problems. For instance, Marinaki et al. [12] proposed Multi-Objective Differential Evolution (MODE) to optimize a fuzzy control system for suppressing vibrations in smart structures. From their experiments, DE was shown to be effective in dealing with control design problems that have many and possibly conflicting objectives. The problem of multiple objectives is significant in trajectory tracking since minimizing tracking errors might demand more control activities, hence more energy expenditure. In another study, Subraj et al. [13] proposed a fuzzy-controlled diversity-based DE for noisy multi-objective optimization problems, showing the effectiveness of fuzzy approaches in enhancing DE performance.

Other evolutionary algorithms have been used for optimization-based fuzzy control in trajectory-tracking systems. Tolossa et al. [14] carried out research on FLC for trajectory tracking of a two-wheeled differential-drive mobile robot through PSO for optimizing parameters of the membership function and GA for automated rule base reduction. The results obtained through their simulation and experimentation proved that through optimization, more accurate tracking can be achieved while at the same time reducing the control efforts. Several other recent studies have been able to show how DE-based fuzzy control can be applied in complicated engineering systems. For instance, Ouda and Alavifard [15] developed an adaptive fuzzy-DE method for optimizing damping and inertia coefficients of virtual synchronous generators. Wang et al. [16] designed a robust fuzzy control method for trajectory tracking of autonomous vehicles using multi-objective DE. Chen et al. [17] presented a modified constrained DE algorithm to optimize an interval Type-2 fuzzy Proportional, Integral, And Derivative (PID) controller for hydraulic actuators of humanoid robots.

Even with all these advances, there still exist some research areas that need to be looked into. Quite a number of studies have concentrated their attention on mobile robots and planar trajectory tracking; however, complex three-dimensional tracking systems have not been given much attention. Besides, quite a lot of work has concentrated on minimizing tracking error without giving much thought to control effort or energy expenditure, which should also be taken into account. Even though DE has proven to be effective at solving optimization problems and tuning fuzzy controllers, its use in the field of fuzzy controller optimization for missile three-dimensional tracking has not yet received adequate attention.

Therefore, the current research aims to design an optimal fuzzy controller for three-dimensional missile tracking through the DE algorithm. Fuzzy membership function parameters are considered as design variables, and they are optimized automatically using the DE algorithm. The objective function evaluates both tracking error and control effort so that the tradeoff is made between the two aspects. Performance of the developed controller is analyzed based on tracking error, control effort, and dynamic response, and compared with other optimization and control methods, such as GA.

The rest of this paper outlines the systematic design and analysis of the proposed fuzzy control scheme. First, the mathematical formulation and state-space model of the nonlinear tracking system are derived. The performance of a traditional PID controller is analyzed as a baseline, and afterwards a Mamdani fuzzy controller with optimized control rules using a GA is designed. Finally, a MODE method is used to find the optimum output torque membership functions of the fuzzy control system by taking into account both the tracking error and the control effort simultaneously. These Pareto optimal results are studied in terms of their dynamic response and are compared to the conventional PID and GA-based fuzzy controllers.

2 | Dynamic Modeling of the Tracking System

2.1 | System Structure and Reference Frame

The system that is taken into account in this research is a satellite with two degrees of rotational freedom, the motion of which is determined by two rotations along two different axes. The configuration structure of the satellite and the reference frame used are shown in *Figs. 1* and *2*. Two generalized coordinates of the system are referred to as (θ) and (α), while two control torques are denoted as (M_1) and (M_2). The center of mass of the satellite is situated at the origin of the reference frame.

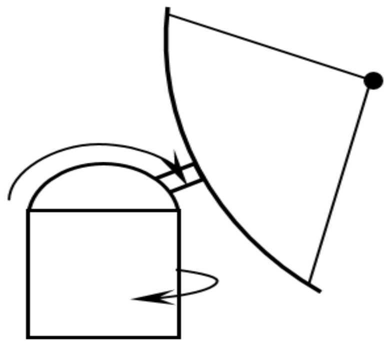

Fig. 1. Schematic of the satellite and its rotation directions.

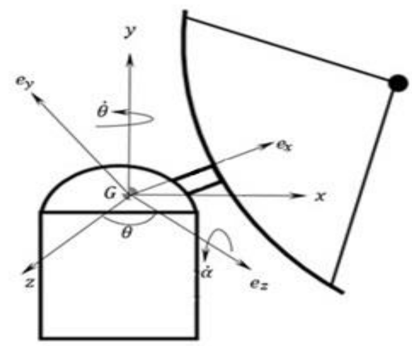

Fig. 2. Schematic of the satellite and the body-fixed coordinate system.

2.2 | Derivation of the Equations of Motion

In order to obtain the equations of motion, the Lagrange equations are used. They have the following general form:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)-\frac{\partial L}{\partial q}=\delta Q, \tag{1}$$

where L = T - U, T is the kinetic energy, U is the potential energy, Q is the differential work, and q is the generalized coordinate. Due to the immobility of the center of mass, the variations of the potential energy are equal to zero (U = 0), and hence L = T.

2.3 | Calculation of Kinetic Energy

For calculating the kinetic energy, an equivalent rod structure is assumed as in *Fig. 2*. Considering an observer on the first rod, there is only the angular velocity $\dot{\theta}$, and the rotational movement about the e_3 axis does not affect it. Since the inertia matrix of the first rod is expressed as I_1 = diag(I_{1x}, $I_{1\gamma}$, I_{12}), its kinetic energy is as follows:

$$T_1=\frac{1}{2}I_{yy_1}\dot{\theta}^2. \tag{2}$$

For the second rod, the observer observes the angular velocity $\dot{\alpha}$ and also $\dot{\theta}$. Since the inertia matrix of the second rod is expressed as I_2=diag(I_{2x}, $I_{2\gamma}$, I_{22}), its kinetic energy can be calculated from the following equation:

$$T_2=\frac{1}{2}[I_{xx_2}(\dot{\theta}\cos\alpha)^2+I_{yy_2}(\dot{\theta}\sin\alpha)^2+I_{zz_2}\dot{\alpha}^2+(I_{yx_2}+I_{xy_2})\dot{\theta}^2\cos\alpha\sin\alpha \\ +((I_{zx_2}+I_{xz_2})\dot{\theta}\cos\alpha+(I_{zy_2}+I_{yz_2})\dot{\theta}\sin\alpha)\dot{\alpha}]. \tag{3}$$

Hence, the total kinetic energy of the system will be calculated by adding the kinetic energies of the two rods:

$$T = T_1 + T_2. \quad (4)$$

The change in the potential energy of the system is equal to zero since the center of mass of the system is fixed at the origin of the coordinate system, and therefore there is no change in altitude. The Lagrangian of the system is equal to the kinetic energy of the system. Upon substitution of (T) and (U) in the Lagrange equation, the governing dynamic equations of the system can be found.

2.4 | Derivation of Equations of Motion by Using the Lagrange Equations

By substitution of the generalized coordinates q = θ and q = α in *Eq. (1)*, the equations of motion for each degree of freedom will be found.

2.4.1 | First case: Substitution of q = θ

Upon substitution of q = θ in *Eq. (1)*:

$$\begin{aligned}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) &= \frac{1}{2}\frac{d}{dt}[2I_{xx_2}\dot{\theta}\cos^2\alpha + 2I_{yy_2}\dot{\theta}\sin^2\alpha + 2(I_{yx_2} + I_{xy_2})\dot{\theta}\cos\alpha\sin\alpha,\\ &((I_{zx_2} + I_{xz_2})\cos\alpha + (I_{zy_2} + I_{yz_2})\sin\alpha)\dot{\alpha} + 2I_{yy_1}\dot{\theta}].\end{aligned} \quad (4)$$

$$\frac{\partial L}{\partial q} = 0. \quad (5)$$

$$\delta Q = M_1\theta, \quad (6)$$

where

$$\begin{aligned}&I_{xx_2}\left(\ddot{\theta}\cos^2\alpha - 2\dot{\theta}\dot{\alpha}\cos\alpha\sin\alpha\right) + I_{yy_2}\left(\ddot{\theta}\sin^2\alpha + 2\dot{\theta}\dot{\alpha}\cos\alpha\sin\alpha\right),\\ &+(I_{yx_2} + I_{xy_2})(\ddot{\theta}\cos\alpha\sin\alpha + \dot{\theta}(\dot{\alpha}\cos^2\alpha - \dot{\alpha}\sin^2\alpha))\\ &+(\frac{1}{2}(I_{zx_2} + I_{xz_2})\dot{\alpha}\cos\alpha + \frac{1}{2}(I_{zy_2} + I_{yz_2})\dot{\alpha}\sin\alpha)\dot{\alpha}\\ &+(\frac{1}{2}(I_{zx_2} + I_{xz_2})\cos\alpha + \frac{1}{2}(I_{zy_2} + I_{yz_2})\sin\alpha)\ddot{\alpha} + I_{yy_1}\ddot{\theta} = M_1.\end{aligned} \quad (7)$$

2.4.2 | Second case: Substituting q=α

By substituting q = α into *Eq. (1)*, we have:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{1}{2}\frac{d}{dt}\left[2I_{zz_2}\dot{\alpha} + \left(I_{zx_2} + I_{xz_2}\right)\dot{\theta}\cos\alpha + \left(I_{zy_2} + I_{yz_2}\right)\dot{\theta}\sin\alpha\right]. \quad (8)$$

$$\begin{aligned}\frac{\partial L}{\partial q} = \frac{1}{2}\Big[&-2I_{xx_2}\dot{\theta}^2\cos\alpha\sin\alpha + 2I_{yy_2}\dot{\theta}^2\cos\alpha\sin\alpha + \left(I_{yx_2} + I_{xy_2}\right)\dot{\theta}^2(\cos^2\alpha - \sin^2\alpha)\\ &+ \left(\left(-\left(I_{zx_2} + I_{xz_2}\right)\dot{\theta}\sin\alpha\right) + \left(I_{zy_2} + I_{yz_2}\right)\dot{\theta}\cos\alpha\right)\dot{\alpha}\Big].\end{aligned} \quad (9)$$

$$\delta Q = M_2, \quad (10)$$

where

$$\begin{aligned}&I_{zz_2}\ddot{\alpha} + \frac{1}{2}\left(I_{zx_2} + I_{xz_2}\right)\left(\ddot{\theta}\cos\alpha - \dot{\theta}\dot{\alpha}\sin\alpha\right),\\ &+\frac{1}{2}(I_{zy_2} + I_{yz_2})(\ddot{\theta}\sin\alpha + \dot{\theta}\dot{\alpha}\sin\alpha) + \frac{1}{2}((-I_{xx_2}\dot{\theta}^2\cos\alpha\sin\alpha)\\ &+I_{yy_2}\dot{\theta}^2\cos\alpha\sin\alpha) + \frac{1}{2}(I_{yx_2} + I_{xy_2})\dot{\theta}^2(\cos^2\alpha - \sin^2\alpha)\end{aligned} \quad (11)$$

$$+\frac{1}{2}((-(I_{zx_2}+I_{xz_2})\dot{\theta}\sin\alpha)+\frac{1}{2}(I_{zy_2}+I_{yz_2})\dot{\theta}\cos\alpha)\dot{\alpha}=M_2.$$

2.5 | Conversion of Second Order Equations to State Space Equations

To carry out numerical solution and optimize the system, the second-order *Eqs. (4)* and *(6)* need to be expressed as first-order equations through:

$$\theta=y_1\,,\ \dot{\theta}=y_2\,,\ \alpha=y_3,\dot{\alpha}=y_4. \tag{12}$$

The state-space equations are obtained as follows:

$$y_1=\theta\Rightarrow\dot{y}_1=\dot{\theta}. \tag{13}$$

$$y_2=\dot{\theta}\Rightarrow\dot{y}_2=\ddot{\theta}. \tag{14}$$

$$y_3=\alpha\Rightarrow\dot{y}_3=\dot{\alpha}. \tag{15}$$

$$y_4=\dot{\alpha}\Rightarrow\dot{y}_4=\ddot{\alpha}. \tag{16}$$

The set of *Eqs. (6)-(9)* is the last state space representation of the system, and the same equations will be used for the remaining analysis in this study.

$$\begin{gathered}(7854.81\cos^2\alpha+6461.67\sin^2\alpha+0.915632)\ddot{\theta}-2786.28\dot{\theta}\dot{\alpha}\cos\alpha\sin\alpha=M_1\\ \Rightarrow\ddot{\theta}=\frac{M_1+2786.28\dot{\theta}\dot{\alpha}\cos\alpha\sin\alpha}{7854.81\cos^2\alpha+6461.67\sin^2\alpha+0.915632}.\end{gathered} \tag{17}$$

$$\begin{gathered}6379.30\ddot{\alpha}+\frac{1}{2}((-7854.81\dot{\theta}^2\cos\alpha\sin\alpha)+6461.67\dot{\theta}^2\cos\alpha\sin\alpha)=M_2\\ \Rightarrow\ddot{\alpha}=\frac{M_2+696.57\dot{\theta}^2\cos\alpha\sin\alpha}{6379.30}.\end{gathered} \tag{18}$$

Now, by combining *Eqs. (13)-(16)*, the following equations are obtained:

$$\dot{y}_1=y_2. \tag{19}$$

$$\dot{y}_2=\frac{M_1+2786.28y_2y_4\cos y_3\sin y_3}{7854.81\cos^2 y_3+6461.67\sin^2 y_3+0.915632}. \tag{20}$$

$$\dot{y}_3=y_4. \tag{21}$$

$$\dot{y}_4=\frac{M_2+696.57{y_2}^2\cos y_3\sin y_3}{6379.30}. \tag{22}$$

3 | Control of the System Using PID Controller

At first, the PID controller is applied to the set of equations for the system under consideration in this thesis in order to see the response of the system when an ordinary and suitable controller controls it.

A PID controller is made up of a proportional gain, an integrator, and a differentiator having constant coefficients. With respect to the coefficients of the PID in *Table 1*, the response of the system for the unit step input will be shown in two modes.

Table 1. PID coefficients.

Coefficient	θ	α
Proportional (P)	8	8
Integral (I)	3	3
Derivative (D)	1.1	0.7

In *Fig. 3*, the horizontal axis shows the time while the vertical axis shows the response (error) for the first mode. The first mode includes the values of θ and $\dot{\theta}$.

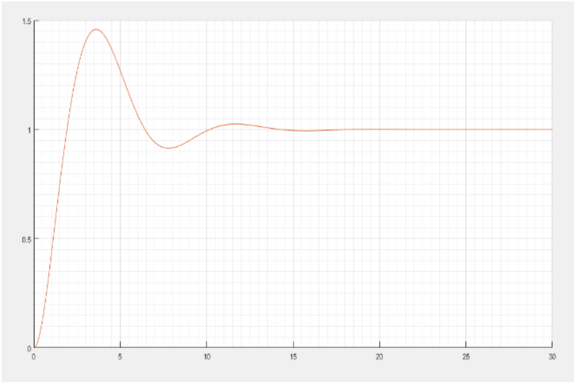

Fig. 3. Response of the system to unit step input for the first mode.

Again, in *Fig. 4*, the horizontal axis shows the time while the vertical axis shows the response for the second mode. The second mode includes the values of α and $\dot{\alpha}$.

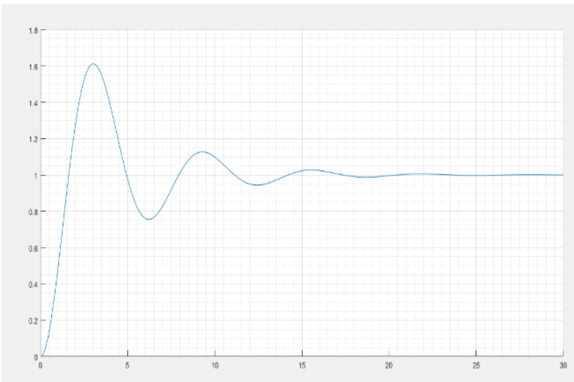

Fig. 4. System response to unit step input for the second mode.

As seen above, the response of the first variable has about 50% overshoot and a settling time of 10 seconds. In the case of the second variable, the response has 60% overshoot and a settling time of 15 seconds. This kind of response is not at all acceptable for a military system where the requirement is to track a target very quickly. Thus, this particular controller, despite all its merits, cannot be used for this system. With the analysis of the response of the PID controller being completed now, the next control strategy to discuss is the fuzzy controller. In a fuzzy controller, some rules as well as membership functions are required, which are normally provided by people who know about the system very well.

However, there is another way of doing it. The system rules can be obtained through optimization, where an optimal chromosome will be generated instead of having to depend on an expert. In fact, the optimization technique, after some time and through many generations, will result in the generation of a chromosome that is able to perform at the same level as the expert designer of the fuzzy controller. The only problem with this approach is that the membership functions of the system have to be initialized. This means that the membership functions do not have any other way other than depending on the initial knowledge of a person familiar with the system.

4 | Optimization using Genetic Algorithm

Before optimizing the torque, it is essential to distinguish the membership functions related to that torque. Torque M_1 has the two variables $y_1 = \theta$ and $y_2 = \dot{\theta}$. In the case of optimizing torque M_1, only the membership functions of those two parameters will be used, as shown in *Fig. 5*.

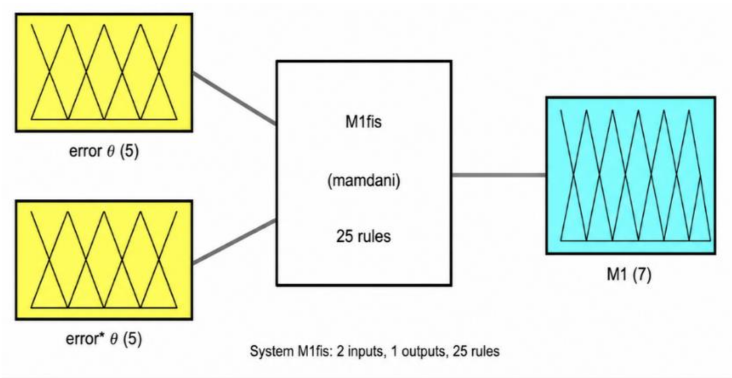

Fig. 5. Relation between torque M_1 and the input membership functions.

The same thing applies to torque M_2, which uses the variable $y_3 = \alpha$ $y_4 = \dot{\alpha}$. This can be seen from *Fig. 6*.

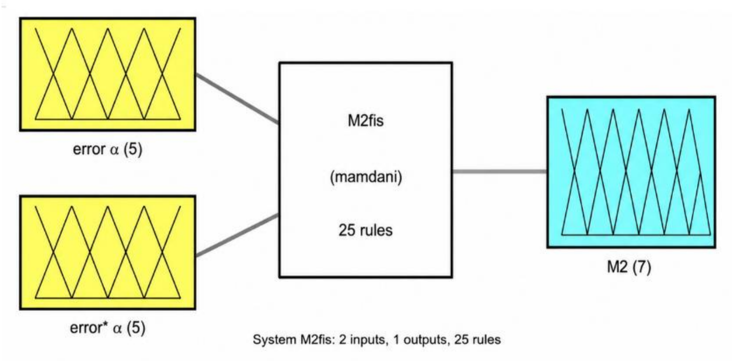

Fig. 6. Relation between torque M_2 and the input membership functions.

In order to choose the right response, one should use an algorithm that solves one problem only, and in which the cost function is designed to minimize the error due to the step response of each of the states α and θ. In the cost function, the error of each state variable is calculated compared to its target value by using numerical integration using the trapezoidal rule. In addition to the complexity of this problem and the lengthy processing time needed to solve it, a small initial population size was selected. However, a mutation operator is included in the algorithm to optimize the current response.

4.1 | Fuzzy Control System Design with Optimized Rules

After analyzing the performance of the PID control system, this part will consider the design of a fuzzy control system. The fuzzy system used is of Mamdani type, having five triangular membership functions for each input and two output torques, namely M_1 and M_2. The complete fuzzy control system has four inputs such that $y_1 = \theta, y = \dot{\theta}, y = \alpha$ and $y_4 = \dot{\alpha}$.

4.1.1 | Design of membership functions

These membership functions have been defined with the help of previous knowledge of the system, and the objective is to make sure that the entire input range is covered (condition of completeness). Due to the same pattern of membership functions for each of the input variables, *Table 2* shows the membership functions along with GA parameters.

Table 2. Membership functions (for variables y1,y2,y3,y4) and GA parameters.

Fuzzy Set Label	Membership Function (μ(x))		GA Parameter	Value
Big Negative (BN)	$y = 2x + 1$ $y = 0$	$x < -0.5$ $x > -0.5$	Initial population size	100
Low Negative (LN)	$y = 2x + 2$ $y = -2x$ $y = 0$	$x < -0.5$ $-0.5 < x < 0$ $x > 0$	Number of generations	250
Zero (ZE)	$y = 0$ $y = 2x + 1$ $y = -2x + 1$	$x < -0.5, x > 0.5$ $-0.5 < x < 0$ $0 < x < 0.5$	Crossover probability	0.8
Low Positive (LP)	$y = 0$ $y = 2x$ $y = -2x + 2$	$x < 0$ $0 < x < 0.5$ $x > 0.5$	Mutation probability	0.3
Positive (BP)	$y = 0$ $y = 2x - 1$	$x < 0.5$ $x > 0.5$	Mutation rate (extractive)	0.004

Description of *Table 2*: The first column depicts the linguistic labels used. The mathematical formulation for each of the membership functions in terms of the variable x is given in the second column. The third and fourth columns show the GA tuning parameter names and their respective values. These functions are identical for all four input variables y_1 to y_4; the only difference is the variable name used. The output membership functions of the torque M_1 and M_2 are taken to be symmetric from –10 to +10 kN·m.

4.2 | Comparison of Controller Performance

In order to perform a quantitative comparison of the optimized fuzzy controller, a comparison with the performance of the classical PID controller is conducted. The coefficients of the PID controller are shown in *Table 3* below:

Table 3. PID controller coefficients.

PID Coefficient	Value for θ	Value for α
Proportional (P)	8	8
Integral (I)	3	3
Derivative (D)	1.1	0.7

Membership functions for the four input variables (y_1) to (y_4) are depicted in *Fig. 7*. The normalized input space is considered over the range ([-1,1]) and consists of five fuzzy sets, namely, BN (Big-Negative), LN (Low-Negative), ZE (Zero), LP (Low-Positive), and BP (Positive). Triangular membership functions with proper overlap have been used to ensure a smooth transition from one fuzzy set to another. The same membership function structure has been considered for all four inputs, with the only difference being in their definitions.

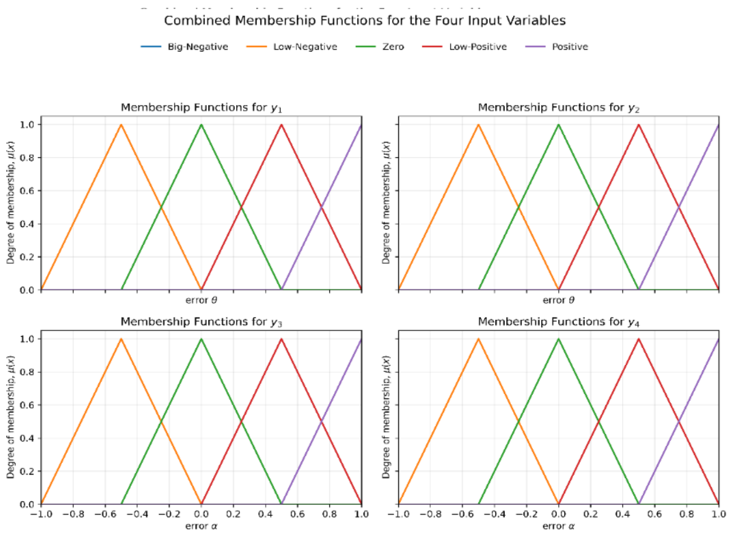

Fig. 7. Membership functions for the four input variables (y_1), (y_2), (y_3), and (y_4).

4.3 | Optimization of Fuzzy Rules by Genetic Algorithm

Now, utilizing the values of the membership functions defined in *Table 4*, the fuzzy rules are optimized by means of a GA. The parameters of the GA are given in *Table 4* below.

Table 4. GA optimization parameters.

Parameter	Value
Initial population size	100
Number of generations	250
Crossover probability	0.8
Mutation probability	0.3
Mutation rate (extractive)	0.004

In the course of optimization, the objective function is designed as the minimization of the Integrated Squared Error (ISE) simultaneously for both states. Being a stochastic method, GA optimization was performed several times with different initial populations of solutions to choose the one providing the minimal objective function value. Dynamic characteristics of the suggested GA-optimized fuzzy controller were calculated and compared with those of the standard PID controller. Step responses of the GA-optimized fuzzy control system for the first and the second operating mode are shown in *Fig. 8*. As can be seen, the proposed controller demonstrates much better dynamic characteristics with much faster transient response, less overshoot, and shorter settling time in both operating modes.

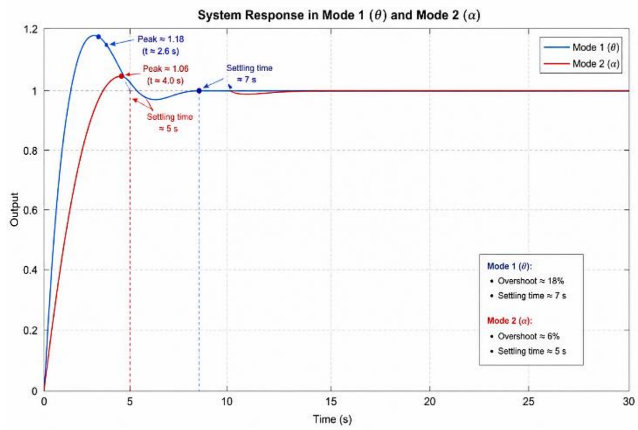

Fig. 8. Step response of the system for the first and second mode (θ and α).

4.4 | Comparison of PID and Fuzzy Controllers

Fig. 9 compares the transient responses of the PID and GA-based fuzzy controllers for the second operating mode (α). It is seen that the classical PID controller has a relatively high overshoot, equal to about 60%, as well as noticeable oscillations before attaining the steady-state value. The required settling time is 15 s.

However, the new fuzzy controller designed using the GA greatly improves the dynamic response of the closed-loop control system. Specifically, the maximum overshoot is minimized to less than 5%, whereas the settling time is less than 5s. Also, the oscillations, which occur with the classical PID controller, are almost entirely suppressed. In other words, the proposed fuzzy controller provides faster and smoother convergence to the reference value. It should be noted that the overshoot reduction provided by the new controller is 55% points, and the settling time reduction is greater than 60% compared to the PID controller.

The same improvement can be seen from the first operating mode (θ) as shown in *Fig. 10* below. The optimized fuzzy controller has a much smaller overshoot and converges to the steady state much faster than the PID controller while still having good tracking performance. This indicates that the proposed control scheme is able to achieve better transient behavior in both operating modes.

From the comparative results shown in *Figs. 9* and 10 above, it is clear that the proposed GA-based fuzzy controller can effectively improve the dynamic behavior of the tracking system. The optimized fuzzy controller has improved settling time, decreased overshoot, and improved damping performance, without needing the exact mathematical model of the plant. Thus, it can be concluded that the proposed GA-based fuzzy control scheme is an effective solution for the satellite attitude tracking problem.

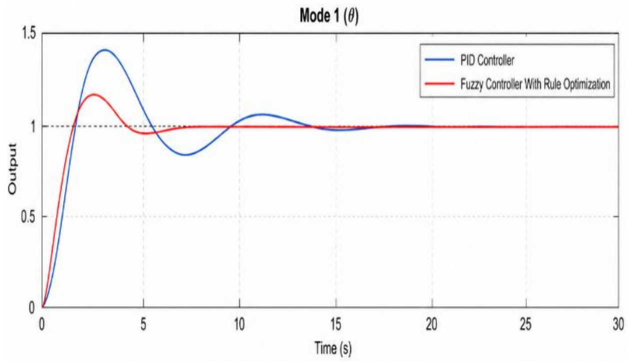

Fig. 9. Fuzzy versus PID controller for first mode (θ).

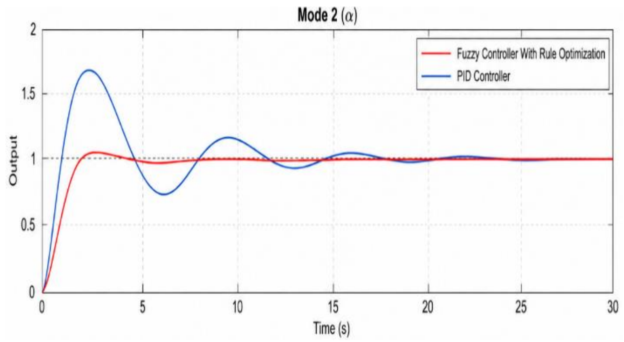

Fig. 10. Fuzzy versus PID controller for second mode (α).

5 | Multi-objective Optimization of Output Torque Membership Functions Using DE

In this work, the input membership functions of the fuzzy controller are supposed to have been optimally designed, which is why only the output membership functions corresponding to the control torques are optimized employing the DE technique.

In this optimization problem, both of the following objective functions are minimized: the tracking error and the control effort (torque). After the completion of the evolutionary process, the solutions that are not dominated in the last generation are considered Pareto optimal solutions. The parameters of MODE algorithm used in this work are listed in *Table 5*.

Table 5. Parameters of the DE algorithm.

Algorithm Parameter	Value
Initial population size	100
Number of generations	300
Crossover rate (CR)	0.5
Mutation factor (F)	1.5
Parent selection	Random
New generation selection	Non-dominated sorting

Using the values given in *Table 5*, the optimization has been done, and the result obtained has been illustrated in the form of the Pareto front in *Fig. 11*.

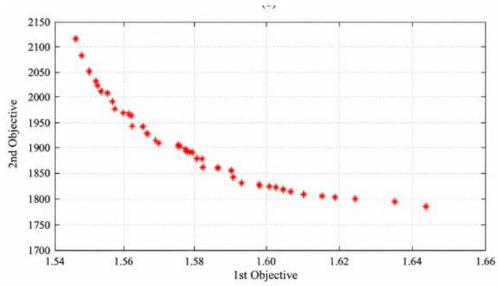

Fig. 11. Pareto front obtained using the DE algorithm.

The step responses of the two operating modes of the system have been analyzed according to the obtained Pareto optimal solutions. These are shown in *Figs. 12* and *13*, for the first and the second operating modes, respectively.

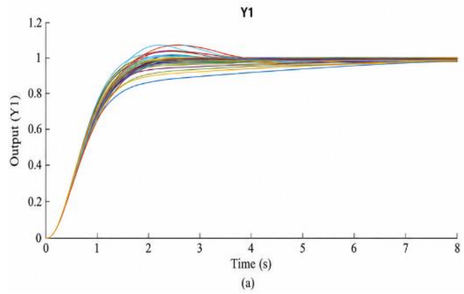

Fig. 12. Step response of the first operating mode using the Pareto-optimal solution for the output torque (M_1).

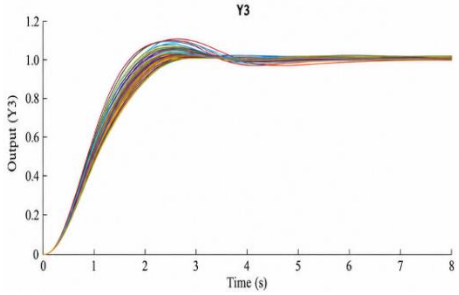

Fig. 13. Step response of the second operating mode using the Pareto-optimal solution for the output torque (M_2).

One Pareto-optimal solution has been chosen among all such solutions to compare with the previous controllers, such as the traditional PID controller and GA-optimized fuzzy controller. The optimized membership functions for the selected Pareto-optimal solution have been shown in *Figs. 14* and *15* for the output torques (M_1) and (M_2), respectively.

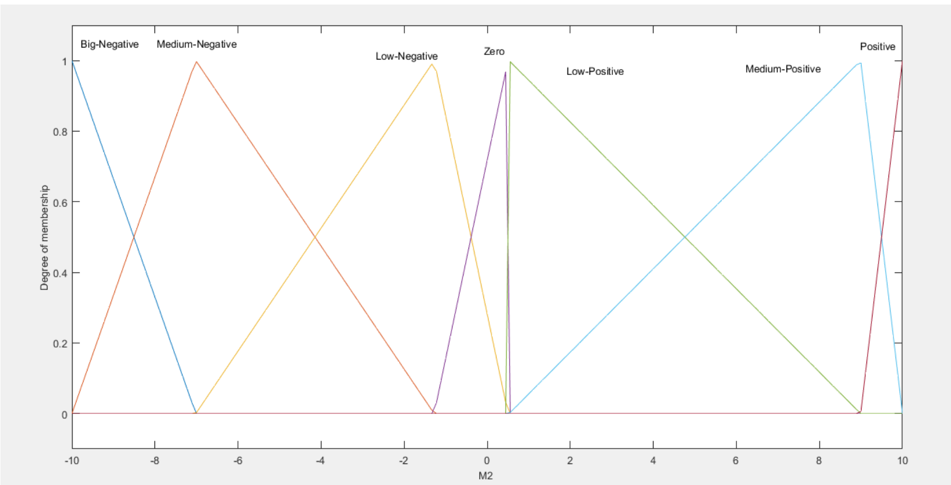

Fig. 14. Optimized torque membership function of the output torque (M_1).

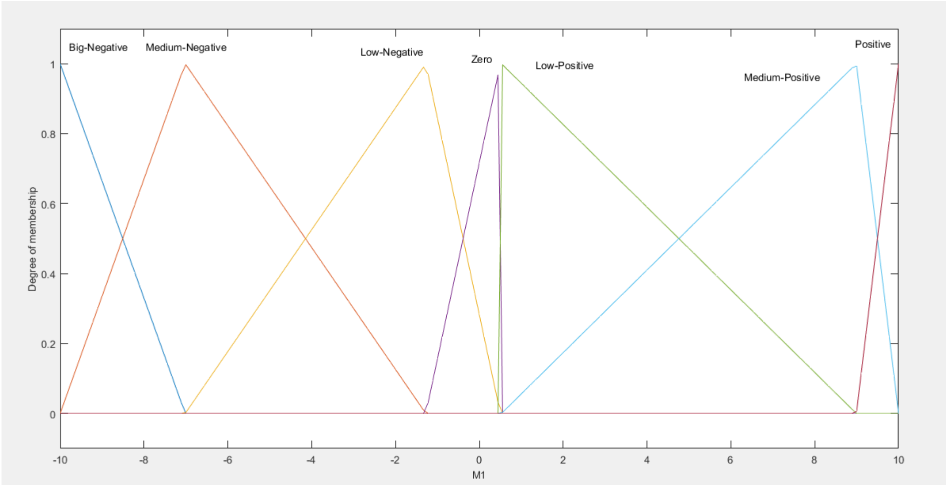

Fig. 15. Optimized torque membership function of the output torque (M_2).

The step responses of the closed-loop system are obtained based on the optimized torque membership functions and compared with the responses of the PID controller and GA-optimized fuzzy controller. The comparative results of the responses for the two operating modes are illustrated in *Figs. 16* and *17*, respectively.

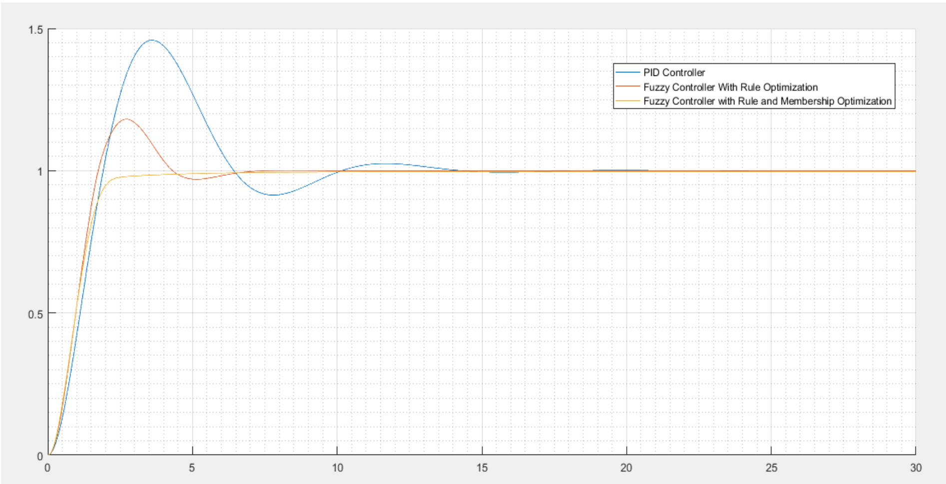

Fig. 16. Comparative responses of the three controllers for the first operating mode.

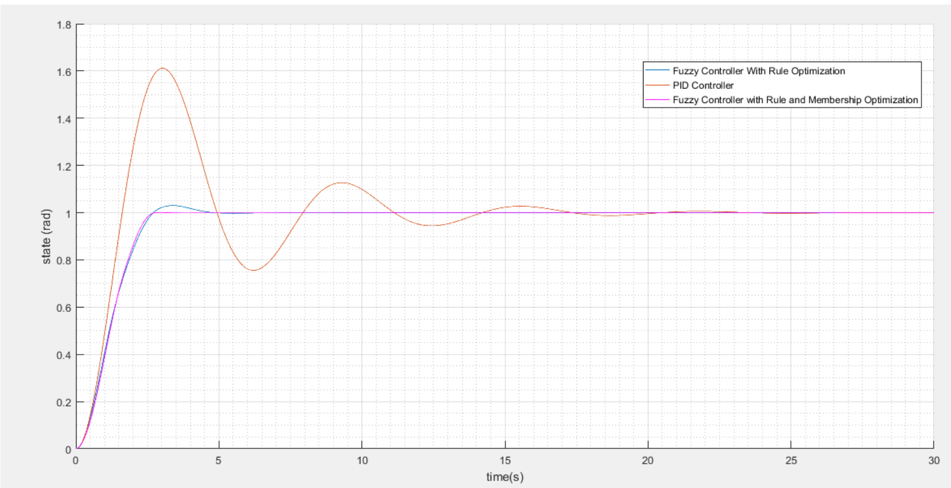

Fig. 17. Comparative responses of the three controllers for the second operating mode.

It can be seen from the figures above that the selected initial torque membership functions ensure adequate control performance. Nevertheless, the optimization with the DE method results in further improvement of the transient response through reduction of the overshoot and settling time. Hence, the proposed DE-based optimization improves the control system performance better than the GA-based one does.

6 | Conclusion

In this work, a FLC approach is proposed for the attitude tracking control of a nonlinear coupled system with two rotational degrees of freedom. The primary aim here was to enhance the tracking performance of the system while minimizing unwanted transients and effort exerted by the control action. Initially, a Mamdani-type FLC system was designed, and its performance was evaluated as compared to the classical PID controller. From the simulation results, it could be seen that the classical PID controller showed considerable overshoots and slow response time for both operating modes. The performance of this controller was thus unsatisfactory for the intended application of attitude tracking. For the first and second operating modes, the overshoot was 50% and 60% with response times of 10 and 15 seconds, respectively.

To address these constraints, a GA was then used to tune the fuzzy control rules. The GA tuned fuzzy controller showed considerable enhancement in the transient response in comparison to the PID controller. In the first operating mode, the overshoot value decreased from around 50% to below 20%, while the settling time dropped from about 10 s to below 7 s. In the second operating mode, the overshoot value decreased from around 60% to below 5%, while the settling time dropped from about 15 s to below 5 s. These results highlight the efficiency of the evolutionary tuning of fuzzy controllers.

Finally, a MODE algorithm was used to optimize the membership functions of the outputs corresponding to the control torques (M_1) and (M_2). Contrary to the previous single-objective optimization, the DE-based optimization considered both the tracking error and the control effort objectives at the same time. Therefore, the obtained non-dominated solutions form the Pareto front, representing various tradeoffs between the tracking error and control torque.

Furthermore, the comparative study of the chosen Pareto-optimal point with respect to the conventional PID and GA optimized fuzzy control methods showed that additional benefits could be attained through the output torque membership functions' tuning process. In addition, the method based on DE improved such control performance characteristics as overshoot and settling time compared to the initially designed membership functions as well as GA based fuzzy control system. Thus, one may conclude that the DE technique represents an efficient tool for fuzzy controller parameters' fine-tuning process while resolving contradictory control objectives at the same time.

On the whole, the suggested multi-objective fuzzy control system based on DE provides an effective approach for solving nonlinear 3D tracking problems in cases when the accuracy and control effort are the control objectives. The obtained simulation results prove that combining fuzzy control and the evolutionary algorithm leads to a significant increase in dynamic performance in comparison to conventional PID controllers and GA-based fuzzy optimization. Therefore, the suggested approach may be considered as an efficient framework for intelligent controller design for aerospace and other nonlinear tracking systems.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability

All data are included in the text.

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