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Analytical Modeling of Ship Motion Control Considering Variable Cargo Mass and Time-Dependent External Forces

Evgeny Leonidovich Pankratov*

Nizhny Novgorod State Agrotechnical University, 97 Gagarin Avenue, Nizhny Novgorod, 603950, Russia; elp2004@mail.ru.

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Abstract

For the analysis of the vessel's stability, maneuverability, and control under different operational conditions, accurate mathematical modeling of the ship's motion is required. Usually, conventional ship-motion models are developed for a given loading condition and thus assume that the total vessel mass and associated inertial properties remain constant during the maneuver under consideration. This assumption may prove restrictive in practical applications, such as cargo loading, unloading, or transfer, especially when external forces and moments are also time-varying. In the present study, an analytical model for the motion and control of a vessel is developed that explicitly accounts for variable cargo mass and time-dependent external forces and moments. The formulation is based on Newton's second law for the vessel, including the mass, the added hydrodynamic mass, the mass moments of inertia, the added moments of inertia of the water, the gravitational and buoyancy forces, the hydrodynamic resistance, and the external disturbing forces and moments. The resulting governing equations allow for the time variation of the principal vessel-motion parameters and provide an analytical framework for evaluating the coupled translational and rotational response of the vessel. Successive integration of the governing equations under prescribed initial conditions yields analytical expressions for translational velocity, displacement, angular velocity, and angular displacement. Parametric analyses are then performed to explore the effects of cargo mass, volumetric displacement, water density, vessel moment of inertia, and added hydrodynamic inertia on the vessel response. The results show that increasing cargo mass increases the roll response for a given heeling moment, while increasing vessel displacement and water density decrease the roll angle due to the increased restoring effect associated with buoyancy. The intrinsic and added moments of inertia of the vessel also significantly affect its rotational response. Also, the temporal results indicate that varying external loading can lead to substantial coupled changes in roll, vertical displacement, and trim angle. The proposed formulation provides a relatively simple analytical framework for studying vessel dynamics under varying loading and environmental conditions. It can serve as a basis for developing more comprehensive variable-mass ship-motion and control models.
Keywords: Ship motion control, Variable cargo mass, Ship maneuvering, Mathematical modeling, Time-dependent external forces, Added hydrodynamic mass, Vessel stability, Analytical solution.

1 | Introduction

The movement and maneuverability of a ship are essential to marine transportation because they directly affect navigation safety, operational efficiency, cargo transport, and the vessel's ability to respond to control commands and environmental disturbances. The interaction of rigid-body inertia determines the dynamic

Corresponding Author: elp2004@mail.ru
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behavior of a ship, including added hydrodynamic mass, hydrodynamic resistance, restoring forces, propulsion and steering forces, and external disturbances from waves, wind, and current. Therefore, mathematical models capable of describing the vessel motion are needed for maneuvering prediction, ship-handling simulation, control-system development, and vessel performance assessment under various operating conditions [1–3].

For several decades, the mathematical modeling of ship maneuvering has been developed. Classical ship-motion models usually consider the vessel as a rigid body and write the equations of motion in terms of translational and rotational degrees of freedom. Depending on the purpose of the analysis, models of three, four, or six degrees of freedom can be used. The main degrees of freedom are surge, sway, heave, roll, pitch, and yaw. More detailed models are particularly useful for studying the effects of load conditions, vertical motion, heel, trim, and environmental disturbances. Recent reviews on ship maneuvering modeling have shown that classical approaches such as the Nomoto, Abkowitz, and Maneuvering Modeling Group (MMG) models remain foundational to modern ship-motion prediction and control [1], [2]. One important development in practical ship maneuvering modeling was the calculation method based on the principal characteristics of the ship's hull, propeller, and rudder, presented by Inoue et al. [3]. Their numerical formulation includes coupled surge, sway, yaw, roll, and propeller-revolution motions and has been applied to several merchant ships. The calculated maneuvering characteristics agreed well with the full-scale trial results.

Importantly, the authors also investigated the effect of loading conditions on ship maneuverability and demonstrated that changes in loading conditions can significantly affect the predicted maneuvering response [3]. This is especially relevant to the present study, in which the cargo mass directly affects a vessel's total mass and loading conditions.Another important progress in ship maneuvering simulation is the MMG approach. Yasukawa and Yoshimura [4] reviewed and introduced the standard MMG method for predicting ship maneuvering. The MMG formulation decouples the hydrodynamic contributions of the hull, propeller, and rudder, providing a systematic framework for calculating the corresponding hydrodynamic forces and moments. The standard method was validated using the KVLCC2 tanker, and the authors showed that the approach can provide useful predictions of ship maneuvering behavior at both model and full scales [4]. The MMG methodology has since become one of the main approaches to mathematical maneuvering simulation due to its physically interpretable representation of the individual components contributing to vessel motion.

Fossen [5] also established a general mathematical framework for marine vehicle dynamics. He derives the equations of motion for marine vehicles in terms of generalized position and velocity vectors, including rigid-body inertia, added mass, Coriolis and centripetal terms, hydrodynamic damping, restoring forces, and external environmental forces. This formulation provides a convenient basis for guidance, navigation, and control applications, as the vessel's physical dynamics can be directly related to control inputs and external disturbances [5]. The framework is especially helpful in constructing general mathematical models that incorporate several physical effects simultaneously.Although detailed mathematical models can provide a complete description of the vessel dynamics, determining the hydrodynamic coefficients remains challenging. Traditional approaches depend on captive model experiments, free-running tests, or full-scale sea trials. They can provide useful information but may require significant time and financial resources. Thus, system identification techniques have been increasingly used to estimate the hydrodynamic coefficients required by mathematical maneuvering models [6–8].

Araki et al. [6] investigated the estimation of maneuvering coefficients from experimental, system-based, and CFD free-running trial data using system-identification techniques. Maneuvering coefficients were estimated using extended Kalman filtering and a constrained least-squares approach. Their study has shown the possibility of integrating system identification with experimental and computational data to obtain maneuvering coefficients without relying solely on extensive captive model testing [6]. This approach is particularly important because it offers a bridge between physics-based mathematical models and computational or experimental data. Revestido Herrero and Velasco González [7] proposed a two-step identification methodology for nonlinear ship maneuvering models. In the first stage, the model structure and the uncertain parameters were identified. In the second stage, the initial estimates were refined using a nonlinear prediction-error method together with an unscented Kalman filter. They applied their methodology to a high-speed trimaran ferry using both full-scale and simulated data. The results showed that nonlinear system identification techniques can improve the representation and validation of ship maneuvering models, especially when the vessel response cannot be adequately represented by a simple linear formulation [7].

Sutulo and Guedes Soares [8] also investigated the identification of mathematical ship maneuvering models from free-running tests. Their approach consisted of estimating model parameters by minimizing differences between reference and reconstructed vessel motion histories. These methods illustrate the importance of reliable parameter estimation when using mathematical models to reproduce actual ship behavior. The increasing use of system identification has therefore helped develop more accurate maneuvering models while reducing the dependence on extensive experimental testing. Another important aspect in mathematical ship maneuvering models is parameter identifiability. When several model parameters have similar effects on the predicted response, then the parameters can become strongly correlated, making it difficult to estimate them individually. Luo [9] studied the parameter identifiability of system-based ship maneuvering models and proved that parameter correlations significantly affect the reliability of the estimated hydrodynamic coefficients. The study showed the need for an appropriate model structure and sufficient information on the excitation conditions to ensure reliable parameter estimation.

Later, the design of excitation signals was studied to improve the identification of nonlinear maneuvering models. The optimal design of excitation signals for identification of a nonlinear ship maneuvering model was studied by Wang et al. [10]. It was shown that properly designed excitation signals can increase the information content of the measured response and reduce the uncertainty of the estimated model parameters. These studies show that the accuracy of the ship-motion models depends not only on the governing equations but also on the physical conditions and the parameter variations used in the model identification process. More recently, mathematical ship models have been increasingly combined with data-driven approaches. Dong et al. [11] presented a math-data-integrated model for estimating ship maneuvering motion. Their methodology combined a mathematical model with a data-driven model to compensate for residual prediction errors. The results showed that combining physical knowledge with data-driven techniques can improve predictive performance while preserving the physical interpretation provided by the mathematical model. This trend represents an important development in modern ship motion prediction and demonstrates the continued relevance of physically based mathematical formulations as the basis for more sophisticated prediction methods.

Furthermore, a recent review by Xu and Guedes Soares [12] demonstrated the evolution of mathematical maneuvering models and system identification methods for surface ships. The review covered classical maneuvering models such as the Abkowitz, MMG, and Nomoto formulations; various experimental tests and sea trials; parameter estimation methods; and environmental disturbances. The authors claimed that the reliability of mathematical maneuvering models depends heavily on the chosen model structure and the quality of the available experimental or numerical data [12]. Despite the extensive development of ship maneuvering models, an important practical problem remains: Variations in vessel loading conditions. In actual marine transportation, cargo may be loaded, unloaded, transferred, or redistributed during the vessel's operating life. Such changes affect the vessel's total mass and may also affect its center of gravity, moments of inertia, draft, trim, buoyancy, restoring characteristics, and hydrodynamic response.

Inoue et al. [3] demonstrated the importance of loading conditions to maneuverability. Later maneuvering and modeling studies continued to consider vessel loading and operating conditions as important factors affecting ship dynamics [1], [4]. Most mathematical maneuvering models, however, are derived for a given loading condition in which the principal vessel parameters are assumed constant during a maneuver. This assumption is reasonable for the analysis of short-duration maneuvers under a fixed loading condition. However, it is restrictive if the aim is to study the vessel behavior under continuously changing cargo conditions. In those cases, the vessel mass and possibly its inertia characteristics become time-dependent quantities. Therefore, a mathematical formulation that allows explicit variation of cargo mass can more generally model ship motion under realistic operational conditions.When the external forces and moments are also time-dependent, the problem becomes even more important. The wave-induced forces, hydrodynamic resistance, and other environmental disturbances may be time-varying.

Meanwhile, the mass of the vessel may be varied simultaneously by loading and unloading operations. The resulting System is therefore described by time-dependent parameters and possibly nonlinear interactions between vessel motion, external forces, and loading conditions. A model that incorporates these effects can be a useful analytical tool for studying a vessel's dynamic response under varying operational conditions. This paper addresses this issue by proposing a generalized mathematical model of vessel motion that explicitly accounts for changes in cargo mass. The formulation is based on Newton's second law and considers the vessel's translational and rotational motions simultaneously. It also considers the added masses and added moments of water. Also included in the governing equations are the gravitational and buoyancy effects, the hydrodynamic resistance forces, the external disturbing forces, and their respective moments. A special feature of the proposed formulation is that the cargo mass and the selected external forces or moments can be treated as variables rather than fixed parameters.

Then, an analytical procedure is developed to determine the time response of the main motion variables. The translational velocity, vessel position, angular velocity, and angular displacement are obtained by successive integration of the governing equations and specification of initial conditions. The analytical formulation offers a relatively simple way to study the sensitivity of vessel motion to the main physical parameters without performing a computationally expensive numerical simulation for each parameter variation. The main aim of this study is therefore to develop a mathematical framework for ship motion analysis that considers variable cargo mass and time-dependent external forces. The proposed model is used to study the influence of the main parameters on vertical displacement, roll angle, trim angle, translational velocity, and angular velocity. The resulting parametric study provides insight into how changes in vessel loading and external conditions affect its dynamic response.

In this way, the proposed formulation complements existing ship maneuvering models and provides a basis for further development of more comprehensive variable-mass ship-motion and control models. The main contribution of the present study is thus the inclusion of cargo mass as a variable in an analytical vessel-motion formulation, with concurrent allowance for variable external forces/moments. The proposed framework aims to provide a more flexible description of vessel dynamics in the presence of time-varying cargo mass. In contrast, traditional formulations primarily focus on a fixed loading condition. This allows the model to serve as a basis for further studies involving more realistic loading and unloading processes, nonlinear hydrodynamic effects, environmental disturbances, and feedback-based ship motion control.

2 | Method of Analysis

The motion of a vessel can be described based on Newton's second law. In the framework of this law, the change in the projections of the corresponding velocities is described by the following equations.

$$(m + m_x)\frac{dv_x}{dt} = F_{x1} + F_{x2};\ (L_{xx} + L_x)\frac{d\omega_x}{dt} = M_{kr} - \rho g V x_1 + \sum_{n=1}^{N} P_n x_{2n} + M_{x1} + M_{x2},\ (m + m_y)\frac{dv_y}{dt} = F_{y1} + F_{y2};\ (L_{yy} + L_y)\frac{d\omega_y}{dt} = M_{diff} - \rho g V y_1 + \sum_{n=1}^{N} P_n y_{2n} + M_{y1} + M_{y2}. \tag{1}$$

$$(m + m_z)\frac{dv_z}{dt} = \rho g V - F_g - \sum_{n=1}^{N} P_n + F_{z1} + F_{z2},$$

$$(L_{zz} + L_z)\frac{d\omega_z}{dt} = M_z - \rho g V z_1 + \sum_{n=1}^{N} P_n z_{2n} + M_{z1} + M_{z2},$$

where $dx/dt = v_x$; $dy/dt = v_y$; $dz/dt = v_z$; $d\theta/dt = \omega_x$; $d\psi/dt = \omega_y$; $d\varphi/dt = \omega_z$; m is the vessel mass; x and y are the horizontal movements of the vessel; z is the vertical movement of the vessel; ψ is the vessel trim angle; θ is the vessel heel angle; m_x, m_y and m_z are the attached mass of water; L_x, L_y and L_z are the attached moments of water;

ω_x, ω_y and ω_z are the projections of angular velocity on axes Ox, Oy and Oz; v_x, v_y and v_z are the projections of vessel velocity on similar axes; ρ is the water density; g is the acceleration due to gravity; V is the vessel volumetric displacement; F_g is the vessel gravity (weight displacement);

$\sum_{n=1}^{N} P_n$ is the total weight of cargo accepted on board; F_{x1}, F_{y1} and F_{z1} are the components of water resistance force to hull movements; M_{x1}, M_{y1} and M_{z1} are the moments of water resistance to hull movements; F_{x2}, F_{y2} and F_{z2} are the components of disturbing force; M_{x2}, M_{y2} and M_{z2} are the moments due to wave action; L_{xx}, L_{yy} and L_{zz} are the central moments of inertia of vessel mass relative to coordinate axes;

$x_1 = x_{1c} - x_{1g}$, $y_1 = y_{1c} - y_{1g}$, $z_1 = z_{1c} - z_{1g}$, x_{1c}, y_{1c} and z_{1c} are the coordinates of the vessel's center, x_{1g}, y_{1g} and z_{1g} are the coordinates of the vessel's center of gravity; M_{kr} is the heeling moment of external forces; M_{diff} is the trimming moment of external forces. Integration of the left and right parts of the equations of *System (1)* over time leads to the following result

$$::v_x = \int_0^t \frac{F_{x1}+F_{x2}}{m+m_x} d\tau;\ \omega_x = \int_0^t \frac{M_{kr} - \rho g V x_1 + \sum_{n=1}^{N} P_n x_{2n} + M_{x1} + M_{x2}}{L_{xx}+L_x} d\tau,$$

$$v_y = \int_0^t \frac{F_{y1}+F_{y2}}{m+m_y} d\tau;\ \omega_y = \int_0^t \frac{M_{diff} - \rho g V y_1 + \sum_{n=1}^{N} P_n y_{2n} + M_{y1} + M_{y2}}{L_{yy}+L_y} d\tau. \qquad (2)$$

$$v_z = \int_0^t \frac{\rho g V - F_g - \sum_{n=1}^{N} P_n + F_{z1} + F_{z2}}{m + m_z} d\tau.$$

$$\omega_z = \int_0^t \frac{M_z - \rho g V z_1 + \sum_{n=1}^{N} P_n z_{2n} + M_{z1} + M_{z2}}{L_{zz} + L_z} d\tau.$$

The initial values of the coordinates and movement speeds, as well as the angles of rotation and the corresponding speeds, were set to zero, which can be achieved by choosing the origin. Repeated integration of the equations of *System (1)* over time allows us to obtain the equations of motion of the vessel in the final form

$$x = \int_0^t (t-\tau) \frac{F_{x1}+F_{x2}}{m+m_x} d\tau;\ \theta = \int_0^t (t-\tau) \frac{M_{kr} - \rho g V x_1 + \sum_{n=1}^{N} P_n x_{2n} + M_{x1} + M_{x2}}{L_{xx}+L_x} d\tau,$$

$$y = \int_0^t (t-\tau) \frac{F_{y1}+F_{y2}}{m+m_y} d\tau;\ \psi = \int_0^t (t-\tau) \frac{M_{diff} - \rho g V y_1 + \sum_{n=1}^{N} P_n y_{2n} + M_{y1} + M_{y2}}{L_{yy}+L_y} d\tau. \qquad (3)$$

$$z = \int_0^t (t-\tau) \frac{\rho g V - F_g - \sum_{n=1}^{N} P_n + F_{z1} + F_{z2}}{m + m_z} d\tau.$$

$$\phi = \int_0^t (t-\tau) \frac{M_z - \rho g V z_1 + \sum_{n=1}^{N} P_n z_{2n} + M_{z1} + M_{z2}}{L_{zz} + L_z} d\tau.$$

3 | Discussion

Fig. 1 shows the variation of the ship roll angle, (θ) with the heeling moment produced by external forces, (M_{kr}) for three different cargo masses. The roll angle increases progressively with (M_{kr}) for all cargo loading conditions considered, as shown. The behavior is consistent with the physical increase of rotational tendency under the action of the external heeling moment. A larger heeling moment causes a greater imbalance between the externally applied rotational effect and the vessel's restoring mechanisms under a given loading condition.

This gives a larger heel angle. It is also evident from the figure that the roll response depends clearly on the cargo mass. As the cargo mass increases from 20 to 50 and 80 t, the corresponding curves are shifted to larger roll angles. For a constant external heeling moment, the angular response of a more heavily loaded vessel is, therefore, greater for the range of parameters considered. This behavior shows that cargo loading cannot be treated solely as a static operational parameter; changes in cargo mass can directly affect the vessel's dynamic response. The nearly linear dependence of θ (M_{kr}) shown in the figure is also consistent with the analytical model structure, in which the external heeling moment is directly incorporated into the rotational equation of motion.

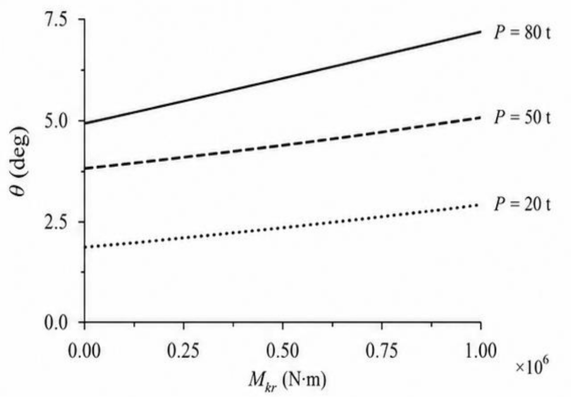

Fig. 1. Typical variation of the ship roll angle, (θ), with the heeling moment of external forces M_{kr}, for different cargo masses (P), $V = 1.0 \times 10^5 m^3$.

Fig. 2 shows the variation of the ship roll angle, (θ) with the volumetric displacement of the vessel, (V) for three different water densities. Unlike in *Fig. 1*, the roll angle decreases as volumetric displacement increases. This tendency indicates that an increase in the displaced volume increases the vessel's resistance to the rotational effect of the prescribed external heeling moment. At a given water density and cargo mass, a vessel with a larger displacement has more restoring effect from buoyancy, which reduces its angular response. The separation of the curves also indicates the effect of the water density. The roll angle is higher for lower water density and lower for higher water density for a given volumetric displacement. The increase in water density enhances the buoyancy force associated with a given displaced volume and consequently modifies the restoring contribution acting on the vessel. So, the response of the vessel depends not only on its geometric displacement, but also on the properties of the fluid surrounding it. The results reported in *Fig. 2* highlight the coupled effect of vessel displacement and fluid density on ship stability. In particular, the simultaneous consideration of (V) and ϱ is important when the proposed model is applied to vessels operating under different environmental or loading conditions. The monotonic decrease in roll angle with increasing displacement also confirms the sensitivity of the developed formulation to the buoyancy-related parameters included in the governing equations.

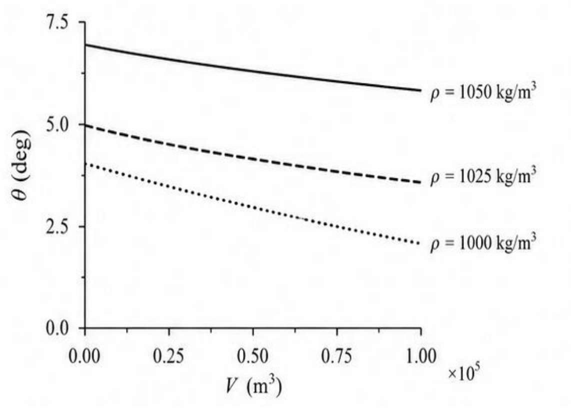

Fig. 2. Variation of the ship roll angle, (θ), with the volumetric displacement, (V), for different water densities, ϱ (($M_{kr} = 5.0 \times 10^5, N, m$), ($P = 50, t$).

Fig. 3 shows the effect of vessel central mass moment of inertia, (L_{xx}), on roll angle, (θ) with three different values of added moment of inertia of water, (L_x). In each of these three cases, there is a downward trend. Thus, the rotational inertia of the vessel-water System increases, reducing the roll response under the given external heeling conditions.

The effect of the added water moment (L_x) is also clearly visible. In the plotted response, larger roll angles correspond to larger (L_x) for a fixed (L_{xx}). Such behavior indicates that the hydrodynamic contribution of the surrounding water has a measurable effect on the vessel's rotational dynamics. The added inertia represents the extra resistance of the surrounding fluid to angular acceleration, thereby altering the vessel's transient rotational response. The differences between the curves for different (L_x) values become smaller as (Lx) increases.

Thus, the effect of the added water inertia diminishes as the vessel's intrinsic rotational inertia becomes the predominant factor. The results therefore stress the importance of considering both the vessel's moment of inertia and hydrodynamic added inertia when modeling roll motion. Neglecting either contribution might lead to an incomplete description of the vessel rotational response.

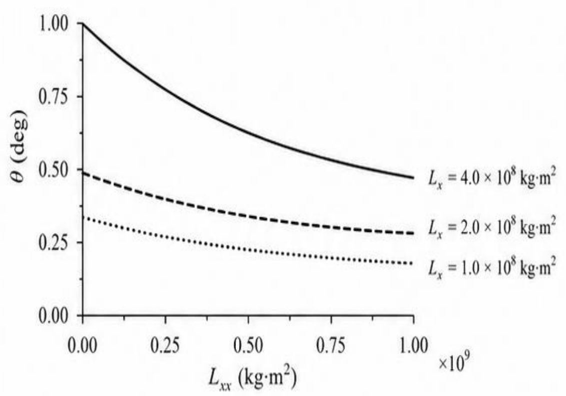

Fig. 3. Variation of the ship roll angle, (θ), with the central mass moment of inertia, (Lxx), for different added moments of inertia of water, $(\mathbf{Lx})((\mathbf{P} = \mathbf{50}, \mathbf{t}), (\boldsymbol{\rho} = \mathbf{1025}, \mathbf{kg/m^3}), (\mathbf{V} = \mathbf{1.0} \times \mathbf{10^5}, \mathbf{m^3}), (\mathbf{M_{kr}} = \mathbf{5.0} \times \mathbf{10^5}, \mathbf{N}, \mathbf{m})$.

Fig. 4 shows the time history of three important motion variables of the vessel: Roll angle (θ), vertical displacement (z), and trim angle (Φ). This figure, contrasted with *Figs. 1-3*, which investigate the model's sensitivity to individual physical parameters, shows the vessel's dynamic response over time under a given set of loading and external-force conditions. The roll angle shows a monotonic increase over the time interval considered. Such behavior indicates that the applied external heeling moment results in a sustained rotational response of the vessel. If there is no compensating control action or a sufficiently strong restoring response, the accumulated rotational effect increases over time. The vertical displacement (z) also increases during the simulation interval, but at a different rate of change than the roll angle. It is the result of the combined action of the vertical forces on the vessel, i.e., gravity, buoyancy, and the prescribed external forces. The trim angle, Φ however, is decreasing with time. The different time evolutions of (θ) and (z)Φ show that the vessel's translational and rotational motions are coupled.

Instead, they are coupled through the mass properties, hydrodynamic added terms, the gravitational and buoyancy forces, and the external moments in the governing equations. Thus, a change in one component of the vessel motion can affect the response in the other degrees of freedom. *Fig. 4* generally shows that the developed formulation can describe time-dependent vessel motion and study the transient response to external forces and moments. The concurrent evolution of roll, vertical displacement, and trim underscores the need to account for coupled translational and rotational dynamics when evaluating the vessel's behavior under different operational conditions.

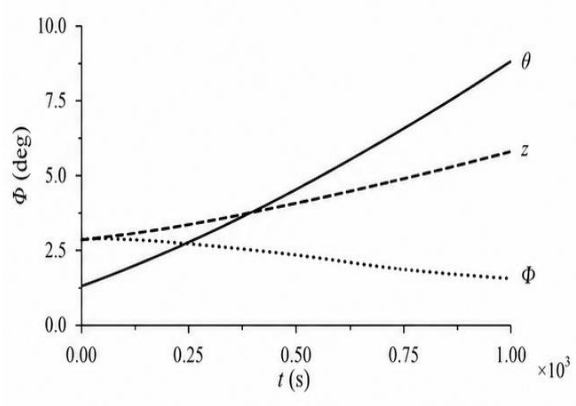

Fig. 4. Typical time histories of the ship roll angle, (θ), vertical displacement, (z), and trim angle, (Φ), under the prescribed external loading condition $\mathbf{M_{kr}} = \mathbf{5.0 \times 10^5 N.m}$ $(\mathbf{P = 50t}), (\boldsymbol{\rho} = \mathbf{1025 kg/m^3})$. $(\mathbf{V = 1.0 \times 10^5, m^3})$).

4 | Conclusion

An analytical mathematical model was developed to study vessel motion and control under conditions of variable cargo loading and time-dependent external forces and moments. The proposed framework explicitly allows the cargo mass and the chosen external loads to be time-dependent. In contrast, standard ship-motion formulations typically assume the vessel mass is constant during a maneuver. The model accounts for the main physical mechanisms governing the vessel's motion: gravity and buoyancy, hydrodynamic resistance, added hydrodynamic mass, the vessel's moments of inertia, the water's added moments, and external disturbing forces and moments. The analytical formulation derived from Newton's second law yields expressions for the main variables of the vessel's translational and rotational motion.

The results show that the developed model can simultaneously describe the vertical displacement, roll motion, trim motion, translational velocity, and angular velocity, and thus can be a useful tool for exploring the dynamically coupled behavior of a vessel under various operating conditions. The parametric results show that the vessel's roll response is strongly affected by cargo mass. The predicted roll angle for a given external heeling moment increases as the cargo mass increases from 20 t to 80 t. The result shows the importance of accounting for variations in loading conditions when evaluating the vessel's stability and dynamic response. The effects of volumetric displacement and water density on roll behavior are also significant. It is shown that the larger the displaced volume or the water density, the smaller the roll angle, thereby establishing the important contribution of buoyancy-related restoring effects to the vessel's rotational response under the conditions studied. Examination of the rotational inertia parameters reveals that the vessel's central mass moment of inertia and the added moment of inertia of the surrounding water significantly influence the roll response. The added hydrodynamic inertia modifies the rotational dynamics.

It is considerable when the inertia of the vessel is relatively small, whereas the intrinsic rotational inertia of the vessel is augmented to lessen the angular response. The results here support the need to account for both rigid-body and hydrodynamic inertia when developing analytical models of ship motion. The time-dependent analysis also shows that the proposed formulation can represent the simultaneous evolution of different vessel-motion variables. The predicted variations in roll angle, vertical displacement, and trim angle demonstrate that the time histories of translational and rotational motions may differ under the same external loading condition. This behavior highlights the coupled nature of vessel dynamics and the importance of considering interactions among loading conditions, buoyancy, inertia, hydrodynamic effects, and external disturbances. The main contribution of the present study is the development of an analytical framework for vessel motion accounting for both variable cargo mass and variable external forces and moments.

The formulation can serve as a relatively simple tool for a preliminary assessment of vessel dynamic behavior, without requiring a computationally intensive numerical simulation for each loading condition. The proposed model also lays the foundation for future studies on nonlinear hydrodynamic forces, realistic cargo loading and unloading rates, time-varying mass distribution, changes in the center of gravity and moments of inertia, environmental disturbances, and feedback-based motion control strategies. Further development of the model, by including experimentally verified hydrodynamic coefficients and comparing with numerical simulations or full-scale measurements, would improve its applicability to practical ship maneuvering and control problems.

Conflict of Interest

The authors declare no competing interests related to this study. The authors alone are responsible for the content and writing of this manuscript.

Data Availability

All relevant data generated or analyzed during this study are included in this published article. No supplementary datasets are available.

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