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Numerical Investigation of Multistage Toolpath Strategies Enhanced Formability and Thickness Distribution in Single-Point incremental Forming

Farham Khalili[1], Javad Rezapour[1,*]

[1] Department of Mechanical Engineering, Lahijan Brunch, Islamic Azad University, Lahijan, Iran; 2581299444@iau.ir; Javad.rezapour@iau.ac.ir.

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Abstract

This study investigates the effect of multistage toolpath strategies on formability, thickness distribution, and dimensional accuracy in the multistage single-point incremental forming of aluminum sheets. The primary objective was to enhance formability, mitigate localized thinning, and enable the fabrication of conical parts with large wall angles. To this end, three-dimensional finite element simulations were performed using Abaqus/Explicit and validated against experimental results. Spiral toolpaths were developed in MATLAB, and the effects of different multistage forming strategies on strain-path evolution and sheet-thickness distribution were evaluated. In addition, a tensile fracture forming-limit criterion was employed to predict ductile failure and determine the formability limit. The numerical results showed good agreement with the experimental data, with a maximum thickness prediction error of 3.26% and a dimensional error of less than 3% for the final part. The results demonstrated that the multistage strategy alters the strain path from an approximately linear to a nonlinear trajectory and redistributes shear strains along the part wall, thereby reducing stress concentration and improving formability. Consequently, larger wall angles could be achieved without premature failure. Furthermore, incorporating a final corrective stage into the toolpath effectively reduced the undesired bottom bulging and improved the dimensional accuracy of the formed geometry.

Keywords: Single-point incremental forming, Multistage forming, Toolpath strategy, Finite element simulation, Tensile fracture forming limit under tension.

1 | Introduction

Sheet metal forming is one of the important processes in various industries, including the automotive industry, aerospace, medical equipment and part production. The development of products with complex geometries,

Corresponding Author: Javad.rezapour@iau.ac.ir

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low production volumes and the need to reduce manufacturing time and cost has increased the necessity of using more flexible methods compared with conventional die-based processes. In traditional forming methods, the manufacture of dedicated dies usually requires considerable cost and time, and for this reason these methods are never an economical option for rapid prototyping, the production of customised parts and the manufacture of parts with limited production volumes. In this regard, incremental sheet forming, as a novel production technology, has made it possible to form complex parts without the need for a complete and permanent die [1], [2].

In the incremental forming process, plastic deformation is created locally and stage by stage as a result of the contact of the tool with the sheet. In the most common configuration of this process, namely single-point incremental forming or SPIF, the sheet is clamped at its peripheral regions by a blank holder and a rigid tool with a spherical tip moves along a predetermined path. After travelling along each contour path, the tool is displaced by a specified amount in the vertical direction, and, by repeating this motion the sheet gradually approaches the final geometry. The local nature of the contact region between the tool and the sheet makes it possible to apply larger strains than in some conventional forming methods and has turned this process into a suitable option for producing complex, low-volume parts [3].

Despite the good formability and the high flexibility of SPIF, limitations such as local thickness reduction, the initiation of cracks and fracture, geometric deviation, reduced dimensional accuracy and limitations in forming walls with large angles have challenged the widespread application of this process. During forming, the non-uniform distribution of stress and strain can cause the deformation to concentrate in some regions of the sheet. This concentration, especially under conditions in which the part is formed in a single stage and at the final angle, increases the probability of severe thinning and the onset of damage. In addition, the absence of a supporting die in the single-point process can affect the geometric accuracy and the amount of springback of the part [4].

One of the effective solutions for reducing these limitations is the use of multi-stage tool paths. In this method, the final shape of the part is created in several stages and, by using intermediate paths, gradual angles or different depths are produced. Unlike single-stage forming, in which the major part of the deformation is applied in one stage, the multi-stage strategy makes it possible to distribute the strain more appropriately over the surface of the sheet. As a result, the stress concentration is reduced, the thickness reduction is controlled and the probability of fracture occurring in critical regions decreases.

However, the performance of this method depends on the way the tool path is designed, the number of stages, the value of the vertical step, the angle of each stage and the geometric features of the part. An inappropriate selection of the intermediate paths may not only fail to improve formability, but may also increase the process time, create undesired deformations and reduce the dimensional accuracy [5].

Therefore, numerical analysis of the SPIF process can be a suitable tool for investigating the mechanical behaviour of the sheet and for evaluating the effect of different parameters before carrying out practical experiments. By using the finite element method, it is possible to investigate the distribution of stress and strain, the thickness variations, the forming forces, the amount of damage and the conditions for the onset of fracture in different paths. Such an analysis can help to identify the mechanisms affecting deformability and to determine a suitable tool path.

On this basis, in the present research the single-point incremental forming process of metal sheet along multi-stage paths is investigated numerically in order to evaluate the effect of the tool path strategy on the strain distribution, the thickness reduction and the improvement of deformability.

1.1 | Review of the Studies Carried Out

In recent years, single-point incremental forming of metal sheet has attracted considerable attention because of the elimination of expensive dies, its high flexibility and its capability to produce complex parts. Various studies have shown that the behaviour of this process depends on a set of geometric, material and process

parameters and that factors such as the wall angle, the sheet thickness, the tool path, the tool diameter, the friction conditions and the kinematic parameters play a decisive role in the forming limit, the thinning and the surface quality [6-8].

One of the most important topics in SPIF is the investigation of the forming limit and the role of the wall angle. The results of [9] showed that a DC01 steel sheet with a thickness of 2.1 mm can be formed up to an angle of about 80 degrees without fracture. However, this limit is a function not only of the angle but also of the properties of the material, the initial thickness, the tool diameter and the vertical step. Therefore, the evaluation of formability must be carried out within the framework of a multi-parameter interaction [10].

The tool path is another of the most effective factors in controlling the mechanical response of the sheet in the SPIF process. Skjødt et al. [11], through an experimental comparison of helical and stepped paths, showed that, because of the continuity of the tool motion, the helical path produces a more uniform surface and a better surface quality than the stepped path. The results of Zahedi et al. [12] also showed that using a helical path can make the thickness distribution more uniform, because the continuous motion of the tool reduces the local concentration of strain.

On the other hand, Azaouzi and Lebaal [13], using finite element modelling of asymmetric parts, showed that optimising the tool path can improve the thickness distribution by about 7%. Liu et al. [14] also, although they did not report a noticeable difference in the formability limit between the helical and the stepped paths, showed that in the helical path the force curve is more uniform and more stable. Yamashita et al. [15] also stated that even the way in which the tool path is started can affect the final quality of the part and the uniformity of the thickness distribution. Taken together, these studies show that the tool path is not merely a geometric variable, but is a key factor in controlling the strain distribution, the local thinning and the stability of the process.

Alongside the tool path, the initial thickness of the sheet is also one of the most important parameters in the final formability of the part. Ham and Jeswiet [16], by investigating aluminium sheets, showed that increasing the initial thickness of the sheet usually increases the achievable wall angle and improves the forming limit. The results of Golabi and Khazaali [17] on AISI 304 stainless steel also confirmed the same trend.

In the same vein, Ham and Jeswiet [18] and Hussain et al. [19] showed that although increasing the thickness can improve formability, this effect is valid only up to an optimum range and an excessive increase in thickness may not be desirable because of the increase in the process forces and the reduction in efficiency of the system. In more recent studies, Pak et al. [20] also showed, for aluminium alloys, that increasing the initial thickness, in addition to reducing the strain concentration and improving the stability of the process, increases the forces applied to the tool. Therefore, the initial thickness of the sheet must be selected simultaneously in terms of formability, thickness distribution and the capacity of the forming equipment.

The shape and the size of the tool also have a marked effect on the local behaviour of the sheet in the contact region. Pak et al. [20] reported that tools with a smaller diameter in many cases increase formability, because they make it possible to create a more localised and more controlled deformation. Hussain et al. [21] also presented similar results and showed that increasing the tool diameter can lead to a reduction in formability. However, not all studies have reached a single conclusion.

For example, Honarpisheh et al. [22] showed that increasing the tool radius may improve the thickness distribution, whereas Mirnia et al. [23] reported that increasing the tool diameter can increase the process force and reduce the uniformity of the thickness. In addition, Kim and Park [24] emphasised the existence of an optimum diameter for the tool; a diameter at which a suitable balance is established between the strain concentration, the forming force and the stability of the process. These results show that the effect of the tool diameter depends on its interaction with the sheet thickness, the vertical step, the material and the friction conditions.

In the field of friction conditions and lubrication, numerous studies have also been carried out. Kim and Park [24] also investigated the simultaneous effect of the type of tool and of lubrication and showed that the

friction conditions can affect not only the surface quality but also the achievable strain and the time at which fracture starts. In other words, friction in SPIF is not merely a secondary factor in reducing wear; it is also one of the parameters that influence the deformation mechanism and the fracture behaviour of the sheet. Therefore, an accurate analysis of the effect of the friction coefficient, alongside the other process variables, is essential for numerical modelling and for predicting the real behaviour of the sheet.

The effect of the rotational speed and of the feed rate has also been extensively investigated in various studies. Kim and Park [24] showed that increasing the feed rate in many metallic materials can cause a reduction in formability, because the time available for the gradual distribution of strain in the contact region is reduced.

However, Xu et al. [25] reported that optimum values exist for these two parameters and that an excessive increase in either of them does not necessarily lead to an improvement in the process response. McAnulty et al. [26] also showed that under some conditions an increase in the rotational speed can, by increasing the local heat, play a role similar to local hot forming and increase the forming limit. Therefore, the effect of the process speeds cannot be interpreted linearly and independently and it is necessary for these effects to be analysed in relation to the material, the thermal conditions and the characteristics of the contact between the tool and the sheet.

Another important parameter in SPIF is the vertical step of the tool, which directly affects the process time, the forming forces, the thickness distribution and the probability of fracture. Strano [27] and Ambrogio [28] reported that reducing the vertical step usually improves formability and surface quality, because the deformation in each stage is applied more gradually. On the other hand, Duflou et al. [29] showed that increasing the vertical step increases the forming force.

Zahedi et al. [12] also reported that increasing the vertical step can reduce formability by intensifying the stress concentration. However, some results, such as the studies of Petek et al. [30] and Mirnia et al. [23] show that under some conditions increasing the vertical step up to a certain limit can have a positive effect on the thickness distribution. Therefore, as with the other parameters, an optimum value also exists for the vertical step, which depends on the material of the sheet, the initial thickness, the tool diameter and the path strategy.

In some studies, more supplementary parameters such as the direction of tool rotation have also been investigated. Ning et al. [31] showed that the direction of tool rotation has no direct effect on the forming limit, but can affect the process forces and the surface quality.

Vahdati et al. [32] and Jurisevic et al. [33] also reported that the direction of tool rotation, in combination with the rotational speed and the friction conditions, can affect the material flow pattern, the temperature distribution and the stability of the contact. Although this factor is less important than parameters such as the thickness, the tool path or the vertical step in determining the forming limit, it cannot be completely ignored in a comprehensive analysis of the behaviour of the process. Alongside the geometric parameters, the study of forming processes of a similar kind under dynamic loading shows that the use of accurate numerical modelling and advanced coupling methods such as SPH is an efficient tool for accurately predicting the final geometry and the thickness distribution in complex parts [34], [35].

In addition, the use of ductile damage models (ductile damage) and non-linear response analyses alongside parametric prediction algorithms makes it possible to evaluate the stability of the structure and the fracture behaviour in large deformations, which plays a key role in optimising tool path strategies and in improving formability in incremental processes [36-38]. In addition, optimisation approaches based on numerical modelling and advanced algorithms for adjusting the operating variables play a significant role in enhancing the performance of complex engineering systems [39].

1.2 | Research Gap and Novelty of the Present Research

Although previous studies have investigated the effect of some parameters of the single-point incremental forming process, an integrated and analytical understanding of the strain and damage behaviour in multi-stage paths is still limited. Most studies have either focused on numerical analysis or have studied only the effect of

individual parameters and have paid less attention to the simultaneous relationship between the principal stresses, the non-linear strain path and the damage criteria in multi-stage processes.

In addition, the presentation of a reliable analytical model for predicting the forming limit and the damage mechanism, together with simultaneous experimental and numerical validation, is still an important gap in this field. The novelties of the present research are as follows:

I. Presentation of an analytical model based on membrane analysis for extracting the principal stresses in the plane-strain state and in biaxial stretching and for determining the maximum forming angle.

II. Application of the FFL damage criterion for predicting and locating damage in the incremental forming process and comparing it with the experimental and numerical results.

III. Simultaneous investigation of the effect of the key parameters, including the wall angle, the vertical step, the sheet thickness, the tool diameter and the friction coefficient, within a coherent analytical-numerical study.

IV. Analysis and comparison of the strain path in single-stage and multi-stage processes and demonstration of the role of multi-stage strategies in increasing formability and achieving higher angles.

2 | Theoretical Relations and Equations Governing the Problem

The single-point incremental forming (SPIF) process was investigated in this research on the basis of an analytical membrane model. In this model, the contact region of the tool and the sheet was considered as a local plastic region and the principal stresses in the meridional, circumferential and thickness directions were extracted. The material was assumed to be homogeneous, isotropic and perfectly plastic and the effects of the bending moment were neglected in the main analysis. In the plane-strain state, the meridional stress is written as follows [11]:

$$\sigma_{\phi} = \frac{\sigma_{Y}}{1 + \frac{t}{r_{tool}}}, \tag{1}$$

and in the state of equal biaxial stretching:

$$\sigma_{\phi} = \sigma_{\theta} = \frac{\sigma_{Y}}{1 + \frac{2t}{r_{tool}}}. \tag{2}$$

These two relations are the main basis of the stress analysis in SPIF and show that the ratio of the sheet thickness to the tool radius has a direct effect on the stress level and consequently on formability.

The effect of friction in this model was taken into account only in the meridional direction, because the share of the circumferential component in comparison with it is negligible. In general, an increase in friction leads to an increase in the meridional stress and intensifies the thinning. The compact relation for the effect of friction can be expressed as follows:

$$\sigma_{\phi} = \sigma_{\phi B}\exp(k\mu\varphi), \tag{3}$$

where k=1 is for plane strain and k=2 is for biaxial stretching. This relation shows that the greater the friction, the higher the effective stress and consequently the higher the probability of damage. The thinning of the sheet is also one of the most important indices for evaluating formability. In the local region of deformation, the product of the thickness and the meridional stress remains approximately constant, therefore:

$$\sigma_{\phi} t = \text{constant}, \tag{4}$$

and consequently:

$$\sigma_{\phi} = \sigma_{\phi B}\frac{t_0}{t}, \tag{5}$$

It is clear from this relation that a reduction in thickness increases the stress and that this issue is directly related to a reduction in formability. In the geometric analysis, the sine law was also used to estimate the thickness:

$$t = t_0 \sin\lambda, \tag{6}$$

where λ is the angle of the cone surface. The geometric details and its derivation are shown in *Fig. 1*. The maximum achievable wall angle was also obtained from the same relations. The general relation between the stress and the wall angle is summarised as follows:

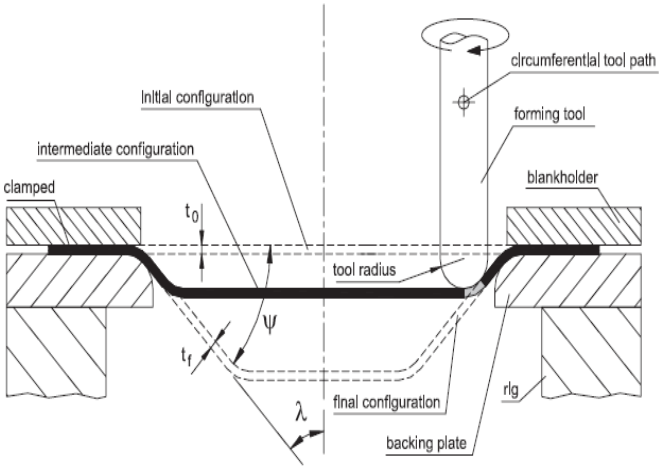


Fig. 1. Representation of the rotationally symmetric single point incremental forming process SPIF [11].

$$\psi = \frac{\pi}{2} - \lambda, \tag{7}$$

and by substituting the relation between the stress and the thickness, the maximum wall angle is qualitatively a function of the initial thickness of the sheet, the tool radius and the stress state. The main result of this analysis is that increasing the sheet thickness and reducing the tool radius increase the achievable angle. For the fracture analysis, the fracture limit diagram FFL was used. In this approach, the accumulation of damage is expressed by the following criterion:

$$D = \int_0^{\varepsilon_f} \frac{\sigma_m}{\bar{\sigma}} d\bar{\varepsilon}, \tag{8}$$

where σ_m is the hydrostatic stress and $\bar{\sigma}$ is the effective stress. Fracture is expected to occur when:

$$D > D_{critical}. \tag{9}$$

In the plane-strain state, the simplified relation of the damage criterion is written as follows:

$$D_{critical} = \frac{1+r}{3}(\varepsilon_{1f} + \varepsilon_{2f}), \tag{10}$$

where r is the anisotropy coefficient. This relation provides the basis for plotting the FFL diagram in the space of the principal strains. Finally, the slope of the fracture limit diagram in SPIF depends on the ratio of the tool radius to the sheet thickness and shows that this diagram, unlike the FLD, must be determined specifically for the incremental forming process and in accordance with the conditions of the tool and the sheet. For this reason, in this research, instead of using the conventional forming diagrams, the FFL extracted from conical and pyramidal specimens was used.

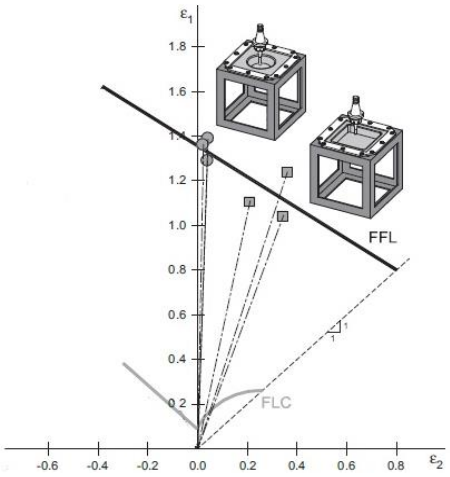


Fig. 2. Schematic of the location of the principal strains in the regions close to the crack in the cone and the pyramid [40].

3 | Numerical Solution Procedure

Single point incremental forming is a strongly non-linear process which, because of the large deformations, the local contact and the thinning of the sheet, can only be simulated in a three-dimensional model. In this research, the numerical analysis was carried out in the Abaqus/Explicit environment so that the behaviour of the sheet in the single-stage and multi-stage processes, together with the effect of the contact and of the tool path, could be investigated accurately. The general schematic of the model, the boundary conditions and the main components of the process are presented in *Fig. 3*.

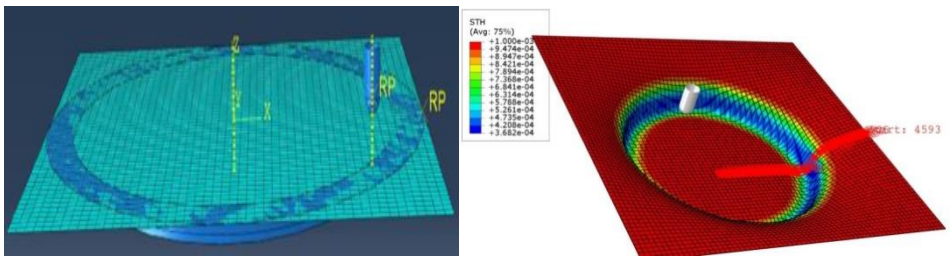


Fig. 3. Simulation of the single point incremental forming process in Abaqus software.

In the numerical model, the forming tool was considered as a rigid body and the AA1050-H111 aluminium sheet as a deformable body. The S4R shell element was used for the sheet, because this element is suitable for analyses involving large deformation and bending-stretching behaviour and is also efficient in terms of computational cost.

By carrying out a mesh convergence analysis, the element size of mm^2 was selected as the optimum state, because a further reduction of the element size did not produce a noticeable change in the thickness response, while the solution time increased considerably. The result of this investigation is shown in *Fig. 4*.

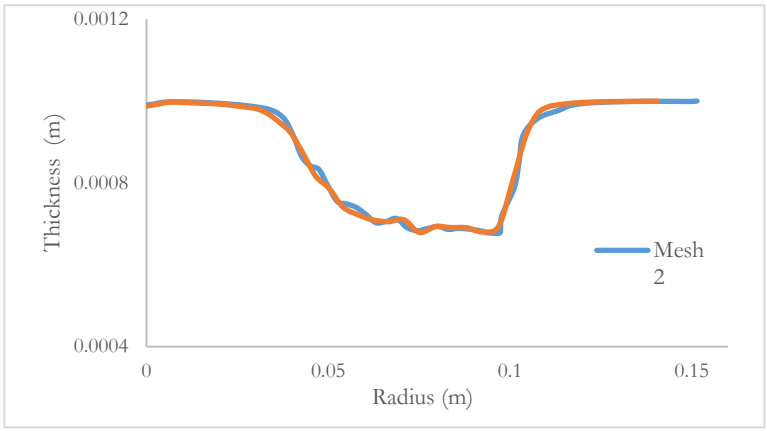


Fig. 4. Investigation of the effect of the meshing on the sheet thickness.

The contact conditions between the tool and the sheet were defined as surface-to-surface and with the Coulomb friction model. In addition, the boundary constraints were applied in such a way that the real behaviour of the process is represented as far as possible. The tool path was generated by coding in MATLAB and the conventional helical path was used to create the final geometry of the part. This method makes it possible to control the vertical step stage by stage and to redefine the path in each stage. The mechanical properties of the sheet used are given in *Table 1*. In addition, in order to evaluate the probability of fracture, the FFLD criterion was used, which in this research was considered in the form of the following relation:

$$\varepsilon_1 + 0.8181\varepsilon_2 = 1.3358. \quad (11)$$

Table 1. Properties of AA 1050-H111 aluminium (Suriyaprakan, 2013).

Property	Value	Unit
Density	2710	Kilogram / Cubic metre
Young's modulus	69	Gigapascal
Poisson's ratio	0.3	-
Yield stress	120	Megapascal

To validate the numerical model, the thickness distribution obtained from the simulation was compared with the results of the experiment. The simulation conditions were considered exactly in accordance with the experimental test, including a 250×250 mm sheet with a thickness of 1 mm, a tool radius of 6 mm, a rotational speed of 1000 rpm and a forming angle of 45°. The comparison of the results showed that the agreement between the numerical and the experimental results is good and that the greatest thickness error was about 3.26%. This comparison is shown in *Fig. 5*.

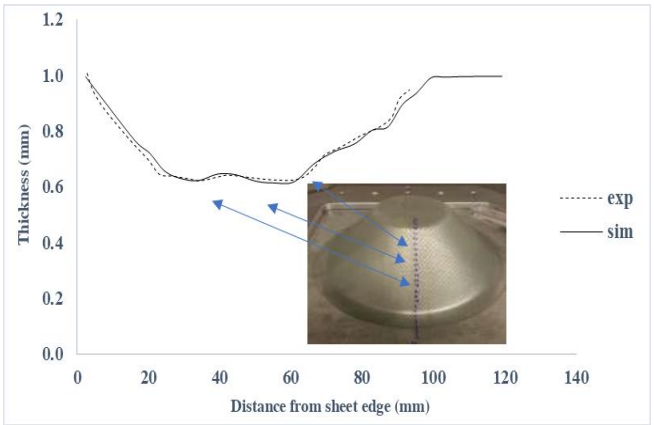


Fig. 5. Comparison of the thickness distribution obtained from the experimental results and from the simulation.

To establish numerical stability and to ensure the correct performance of the explicit solver, the energy balance of the system E_{TOTAL} was monitored; in such a way that the establishment of a balance between the sum of the internal, kinetic and dissipated energies and the external work performed indicates the accuracy of the calculations and the validity of the simulation results.

4 | Discussion and Examination of the Results

In this section, the analytical and numerical results for the single-point incremental forming process of AA1050-H111 aluminium sheet are examined. First, the strain state in the single-stage and multi-stage processes is analysed and then the effect of the forming strategy on the thickness distribution, the strain path, the Fracture Forming Limit (FFL) and the damage criterion is evaluated. Next, an evolved strategy with the aim of reducing the bulge at the bottom, improving the thickness distribution and increasing formability is presented. Finally, the effect of the main parameters of the process, including the initial thickness of the sheet, the tool radius, the wall angle, the vertical step and the friction coefficient, is investigated.

4.1 | Investigation of the Strain State and the Forming Strategies

In the membrane analysis carried out, the material was assumed to be incompressible and consequently the sum of the principal strains was taken to be equal to zero:

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0. \tag{12}$$

The investigation of the principal strains in the critical region showed that the behaviour of the sheet in the single-stage and two-stage processes is close to the plane-strain state, because the value of one of the principal strain components is smaller in comparison with the other two components. As the number of stages increases, the loading path moves away from the plane-strain state and approaches the biaxial stretching region. This change of state results in a more balanced distribution of the strain in the two principal directions, a reduction of the maximum strain concentration and an increase in the formability capacity of the sheet.

To produce a truncated cone with a wall angle of 75°, four strategies including the single-stage process and the two-, three- and four-stage processes were considered. The successive angles in the two-stage process were 45° and 75°, in the three-stage process 45°, 60° and 75° and in the four-stage process 45°, 55°, 65° and 75°. In all the simulations, the dimensions of the sheet were 200×200 mm, its initial thickness 1 mm, the tool radius 6 mm, the rotational speed of the tool 1000 revolutions per minute, the depth of the cone 30 mm and its radius 70 mm.

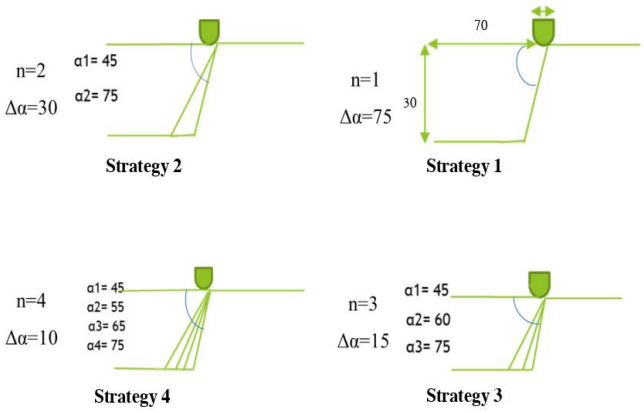


Fig. 6. Combined schematic of the single-stage, two-, three- and four-stage strategies.

The results of the single-stage process showed that direct achievement of a wall angle of 75° causes a severe concentration of deformation at the interface between the bottom and the wall of the cone. In this region,

the thickness of the sheet decreased from the initial value of 1 mm to about 0.4 mm, which is equivalent to 60% thinning. In addition, the critical points passed beyond the FFL line; therefore, the safe production of the intended geometry using the single-stage path was not possible.

The use of multi-stage strategies extended the deformation region and prevented the concentration of thinning within a small area. The minimum final thickness in the two-, three- and four-stage processes was obtained as 0.49, 0.55 and 0.46 mm respectively. Therefore, the three-stage process, with 45 percent thinning, showed the best performance among the initial strategies and reduced the thinning by 15 percentage points in comparison with the single-stage process. Although the two-stage strategy prevented fracture from occurring, the 30° difference between the two stages caused an unbalanced distribution of the thickness and created a bulge at the bottom of the cone.

In the four-stage process, the repeated passage of the tool over the already deformed regions caused the final thickness to decrease in comparison with the three-stage process. Therefore, increasing the number of stages on its own does not guarantee an improvement in formability and the appropriate selection of the sequence of angles and of the tool path is of greater importance.

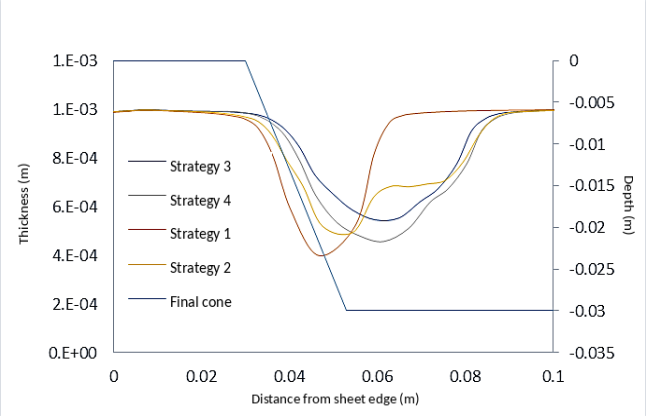


Fig. 7. Comparison of the final thickness distribution in the one-, two-, three- and four-stage strategies.

4.2|Evaluation of the Forming Limit and the Damage Criterion

To evaluate the probability of fracture, the strain path of the critical elements was plotted with respect to the FFL diagram. In this diagram, the location of the strain points below the FFL line indicates safe conditions and their passing beyond this line indicates the onset of damage or tearing. The results showed that the strain path in the single-stage process is almost linear and close to the plane-strain conditions. The maximum and minimum principal strains in the critical element of this process were obtained as 1.44 and 0.057 respectively and the strain path passed beyond the fracture limit line.

In contrast, carrying out the process in a multi-stage manner made the strain path non-linear and transferred the critical points towards the biaxial stretching region. The maximum and minimum strain values in the two-, three- and four-stage processes were respectively (0.856, 0.172), (0.582, 0.236) and (0.72, 0.35)

calculated. In all these strategies, the strain path remained below the fracture limit line; nevertheless, the three-stage process created a safer distance from the FFL line. In multi-stage processes, the change of the angle and of the radius of the tool path means that the critical element of each stage is not necessarily subjected to the greatest deformation in the next stage as well. This displacement of the critical region distributes the strain among a larger number of elements and prevents the accumulation of deformation in one specific region.

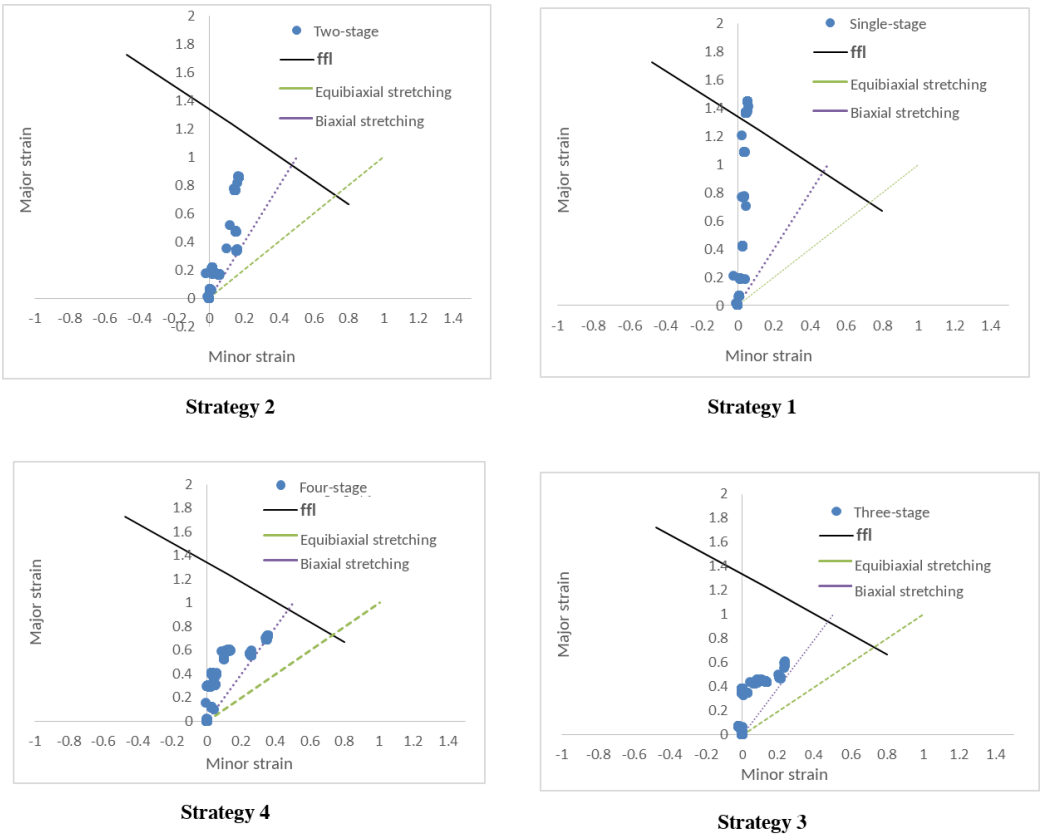


Fig. 8. Combined diagram of the strain path of the single-stage, two-, three- and four-stage processes with respect to the FFLD line.

To quantify the probability of fracture, the damage criterion D was defined on the basis of the ratio of the distance of the strain point from the origin to the distance of the point of intersection of the same direction with the forming limit line:

$$D = \frac{\sqrt{\varepsilon_{min}{}^2 + \varepsilon_{max}{}^2}}{\sqrt{x^2 + y^2}}, \tag{13}$$

where x and y are the coordinates of the point of intersection of the direction passing through the strain point with the FFL line. A value of D<1 indicates safe conditions and D≥1 indicates the occurrence of damage. In the single-stage process, the value of this criterion exceeded one, whereas its maximum values in the two-, three- and four-stage processes were obtained as 0.582, 0.428 and 0.503 respectively.

The lowest value of damage among the initial strategies belonged to the three-stage process. This result is consistent with the thickness distribution and the position of the strain path with respect to the fracture limit line.

4.3 | Evaluation of the Evolved Strategy

Although the three-stage strategy reduced the thinning and the damage criterion, the bulge created at the bottom of the cone limited the geometric accuracy of the part. To solve this problem, an evolved strategy was designed in which the radius and the depth of the cone were increased stage by stage and, in the final stage, a circular helical path was applied at the bottom of the part.

The corrective path of the bottom, with a radius of 50 mm and a step of 4.5 mm, was generated in MATLAB software. In addition to reducing the bulge at the bottom, this path caused the contact of the tool in each stage to be transferred to different regions and the strain concentration in the critical elements to be reduced.

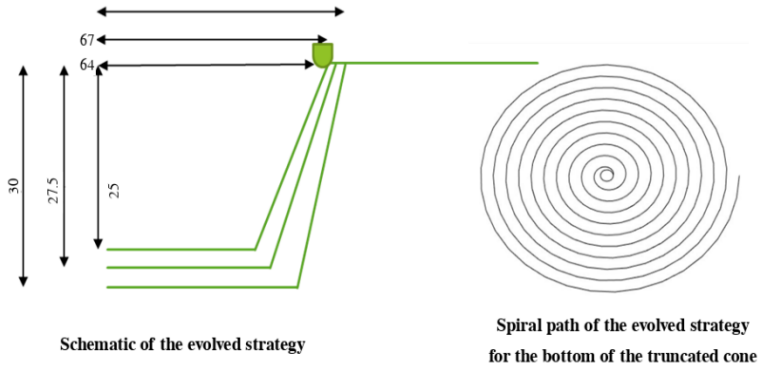


Fig. 9. Schematic of the evolved strategy and comparison of its final geometry with the three-stage strategy.

The minimum sheet thickness in the evolved strategy was obtained as 0.627 mm, whereas this value in the three-stage strategy was 0.55 mm. Therefore, the remaining thickness in the critical region increased by about 14%.

In addition, the maximum damage-criterion value decreased from 0.428 in the three-stage strategy to 0.391 in the evolved strategy, which represents an improvement of about 8.6%. The displacement of the location of maximum damage in this strategy also showed that, as the radius and depth changed in successive stages, the deformation is distributed among different elements.

The comparison of the numerical stresses showed that the meridional stress in the evolved strategy is 59 megapascals, whereas this value in the three-stage strategy was obtained as 81 megapascals. Therefore, the evolved strategy reduced the meridional stress by about 27 percent.

Since this component plays the main role in the stretching and thinning of the wall, its reduction is consistent with the increase of the critical thickness and the reduction in the damage criterion. The numerical and analytical stress values are presented in *Table 2*.

Table 2. Comparison of the meridional and circumferential stresses in the different strategies.

Type of Strategy	Meridional	Circumferential
One stage	96	48
Three stages	81	36
Four stages	38	84
Evolved	59	33
Analytical	102	51

The difference between the analytical and the numerical results of the single-stage process for the meridional and circumferential stresses is about 6.3% and 6.2% respectively, which demonstrates the good accuracy of the analytical model.

Nevertheless, the direct use of the relations based on plane strain is not appropriate for multi-stage processes, because in these processes the strain state gradually approaches biaxial stretching.

Overall, the evolved strategy, by creating a non-linear strain path, reducing the stress concentration and distributing the deformation more appropriately, provided the best performance in terms of formability, remaining thickness, damage criterion and geometric accuracy.

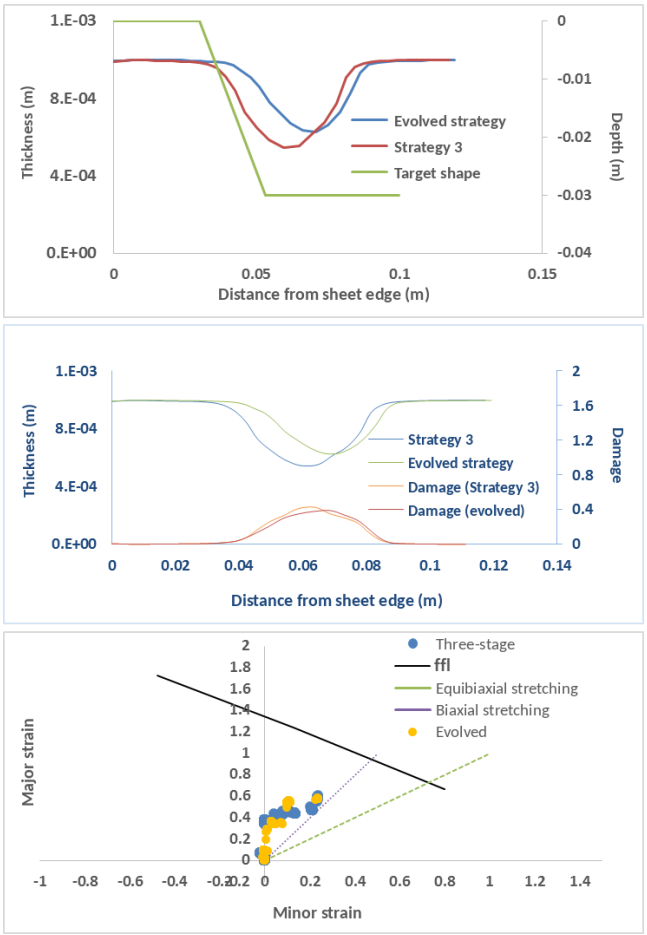


Fig. 10. Combined diagram of the comparison of the thickness, the damage criterion and the strain path in the three-stage and the evolved strategies.

4.4 | The Effect of the Process Parameters on Formability

The effect of the initial thickness of the sheet, the tool radius, the wall angle, the vertical step and the friction coefficient on the remaining thickness in the critical region was investigated. In this analysis, the reduction of the critical thickness was considered as an index of the reduction in formability. The results showed that increasing the initial thickness of the sheet from 1 to 2 mm caused a reduction in relative thinning and an increase in formability. Increasing the initial thickness reduces the meridional stress and makes it possible to achieve a larger wall angle without fracture occurring.

As the tool radius increased from 3 to 7 mm, the remaining thickness in the critical region decreased. A smaller tool applies the deformation within a more limited region and is more compatible with the local nature of the incremental forming process; in contrast, increasing the tool radius increases the contact area and the meridional stress and reduces formability. Increasing the wall angle from 45° to 75° also intensified the thinning. This result shows that the wall angle is one of the most important geometric limitations of the process and that achieving large angles must be carried out through multi-stage paths and the gradual distribution of the deformation.

Increasing the vertical step from 4.2 to 7.5 mm caused a reduction in the critical thickness and an increase of the probability of damage. A smaller step makes the deformation more uniform and more localised, although it increases the process time and the computational cost. In addition, increasing the friction coefficient increased the tensile stress and a reduction in the final thickness. The effect of friction at low values was

limited, but at high friction coefficients the thinning increased noticeably. Therefore, the use of suitable lubrication in order to limit the friction and improve formability is essential.

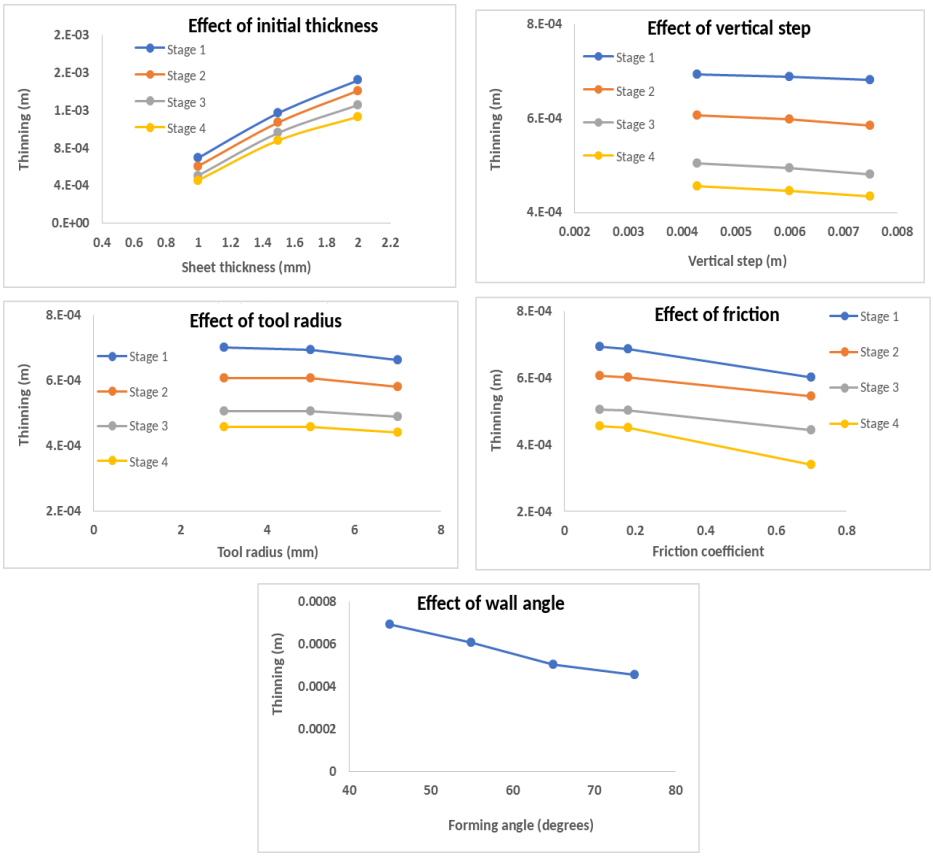


Fig. 11. Effect of the parameters of the sheet thickness, the tool radius, the wall angle, the vertical step and the friction coefficient on the critical thickness.

The overall results of the parametric analysis showed that increasing the initial thickness of the sheet improves formability, whereas increasing the tool radius, the wall angle, the vertical step and the friction coefficient increases the thinning and the probability of damage. Nevertheless, the effect of these parameters is not independent of the forming path and an appropriate selection of the multi-stage strategy can control a considerable part of their undesirable effects.

5 | Conclusion

In this research, the behaviour of single-point incremental forming of AA1050-H111 aluminium sheet, to produce conical parts with steep wall angles, was studied from three aspects: analytical, numerical and experimental. First, using the theory of membrane analysis, a mathematical model was developed for calculating the principal stresses and predicting the maximum forming angle. Then, by implementing the FFL damage criterion in finite element simulations in Abaqus software, the probability of fracture occurring and the thinning of the sheet were evaluated.

To overcome the limitation of single-stage forming at high angles (in particular the angle of 75°), multi-stage tool path strategies and also a modified and evolved strategy were designed and implemented. Finally, by carrying out a parametric study, the effect of the geometric and process variables, including the sheet thickness, the tool diameter, the vertical step, the forming angle and the friction coefficient, on the thickness distribution and the damage criterion was examined, some of which are summarised as follows:

I. Adequacy of the damage criterion: The FFL diagrams showed high efficiency and accuracy in predicting the exact geometric position of the onset of damage and the moment at which tearing occurs in the single-point incremental forming process.

II. Accuracy of the analytical model: The comparison of the principal stresses obtained from the analytical formulation (membrane analysis) with the numerical results in the single-stage process confirmed good agreement; however, because of the deviation of the stress state of the multi-stage processes from the plane-strain conditions, these equations have limitations for the intermediate stages of multiple paths.

III. Change of the stress state in multi-stage processes: The use of multi-stage strategies caused the stress and strain state of the sheet to move away from the plane-strain state and to tend towards biaxial stretching, which strongly increased the formability capacity of the material.

IV. The role of the strain path becoming non-linear: Unlike the single-stage process, which has a linear strain path, the multi-stage strategies, by creating non-linear strain paths and making the deformation more localised in each stage, prevented the accumulation of damage and improved formability.

V. Geometric advantage of the evolved strategy: The application of the evolved strategy (simultaneous change of the depth and the radius together with the final helical path at the bottom of the part), in addition to removing the defect of the bulge at the bottom and improving the dimensional accuracy, increased the critical thickness by about 14 percent and reduced the damage criterion by 8.6 percent in comparison with the conventional three-stage state.

VI. Effect of the geometric and process characteristics: The parametric analysis showed that increasing the initial thickness of the sheet, as a strengthening factor, improves the thickness distribution and reduces the damage, whereas increasing the wall angle, the vertical step, the tool diameter and the friction coefficient has the opposite effect and increases the probability of tearing.

VII. Optimum method of extracting the FFL: The most suitable experimental-numerical methodology for accurately plotting the FFL diagram in this process was determined to be the forming of conical and pyramidal parts with different thicknesses and tool diameters and then connecting the principal strain points corresponding to the moment of fracture in the plane of the principal strains.

Conflict of Interest

The authors certify that they have no affiliations with or involvement in any organization or entity with any financial or non-financial interest in the subject matter discussed in this manuscript.

Data Availability

All data relevant to this study are contained within the manuscript. Additional information is available from the corresponding author upon reasonable request.

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